

CSE 5311.004 Fall 2005

Exercise Set Week 3

(Includes some problems from Weeks 1 and 2)

1. The input is a set S containing n real numbers, and a real number x .
 - a. Design an algorithm to determine whether there are two elements of S whose sum is exactly x . The algorithm should run in $O(n \log n)$ time.
 - b. Suppose now that the set S is given in a sorted order. Design an algorithm to solve the above problem in time $O(n)$.
2. You are given 9 identical looking balls and are told that one of them weighs a bit less than the rest of the eight balls. The only operation you are allowed is to compare a set of balls against another set of balls. Determine the lighter ball using 3 comparisons. Generalize your answer to more than 9 balls if possible.
3. (Harder) You are given 12 balls, and are told that one of them is of a *different* weight from the rest – i.e., you don't know if it is heavier or lighter. Determine this ball using 4 comparisons.
4. Solve the following recurrence equations:
 - a. $T(n) = 2T(n/2) + n \log_2 n$, $T(2) = 4$
 - b. $T(n) = 3T(n/2) + n \log_2 n$, $T(1) = 1$
 - c. $T(n) = T(9n/10) + n$
 - d. $T(n) = T(n - 1) + \log_2 n$
5. Compare the following functions in terms of orders. In each case, say whether $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, and/or $f(n) = \theta(g(n))$
 - a. $f(n) = \sqrt{n}$, $g(n) = (\log n)^5$
 - b. $f(n) = n^2/\log n$, $g(n) = n(\log n)^2$
 - c. $f(n) = \log n$, $g(n) = \log(n^2)$
 - d. $f(n) = n2^n$, $g(n) = 3^n$