CSE 5311.004 Fall 2005

Final Exercise Set

1. Answer true or false to the following questions, and give proofs:
   - A problem X is NP-Complete if for all problems Y in NP, X $\leq_P$ Y
   - If problem X is polynomially reducible to Y and Y is polynomially reducible to Z, then X is polynomially reducible to Z
   - 1-SAT is in P
   - 2-SAT is in NP
2. Given an algorithm A for the decision version of the *Maximum Independent Set* problem, design an algorithm B to compute the *size of the maximum independent set* that uses A as a subroutine. If A takes time T(n, m) where the graph has n vertices and m edges, what is the running time of B?
3. Given two graphs G and G', the *Subgraph Isomorphism* problem seeks to determine whether G' is a subgraph of G if we are allowed to relabel the vertices of G' but make no other changes to its structure. Prove that the *Subgraph Isomorphism* problem is NP-complete.
4. Show that the *Maximum Independent Set* problem is NP-Complete by reducing from *Vertex Cover.* Likewise, prove in the reverse direction, i.e. that *Vertex Cover* is NP-Complete by a reduction from *Maximum Independent Set*.
5. Design a polynomial algorithm for computing an optimal vertex cover for graphs that are trees.
6. The vertex-cover problem and the clique problem are complimentary in the sense that the complement of an optimal vertex cover corresponds to a maximum clique in the complement graph. We also know that the vertex cover problem has an approximation algorithm with a constant approximation ratio. Does that mean that there is an approximation algorithm for the clique problem with a constant approximation ratio? Justify your answer.
7. Solve problem 35.2-3 from the book.
8. Solve problem 35.2-5 from the book.