SET 1

1. The input is a set S containing n real numbers, and a real number x.

   a. Design an algorithm to determine whether there are two elements of S whose sum is exactly x. The algorithm should run in O(n log n) time.

   b. Suppose now that the set S is given in a sorted order. Design an algorithm to solve the above problem in time O(n).

2. You are given 9 identical looking balls and are told that one of them weighs a bit less than the rest of the eight balls. The only operation you are allowed is to compare a set of balls against another set of balls. Determine the lighter ball using 3 comparisons. Generalize your answer to more that 9 balls if possible.

3. (Harder) You are given 12 balls, and are told that one of them is of a different weight from the rest – i.e., you don't know if it is heavier or lighter. Determine this ball using 4 comparisons.

4. Solve the following recurrence equations:

   a. $T(n) = 2T(n/2)+ n\log_2 n$, $T(2) = 4$
   b. $T(n) = 3T(n/2) + n\log_2 n$, $T(1) = 1$
   c. $T(n) = T(9n/10) + n$
   d. $T(n) = T(n – 1) + \log_2 n$

5. Compare the following functions in terms of orders. In each case, say whether $f(n) = O(g(n))$, $f(n) = \boldsymbol{\Omega}(g(n))$, and/or $f(n) = \boldsymbol{\theta}(g(n))$

   a. $f(n) = \sqrt{}n$, $g(n) = (\log n)^5$
   b. $f(n) = n^2/\log n$, $g(n) = n(\log n)^2$
   c. $f(n) = \log n$, $g(n) = \log(n^2)$
   d. $f(n) = n2^n$, $g(n) = 3^n$