Scheduling problem

• Let there be n jobs ($x_n$), each job has a restriction of start time ($s_i$), duration ($d_i$) and end time ($e_i$). Where $e_i - s_i >= d_i$

• Decision Problem:
  – I/O: Set of jobs with $s_i$, $d_i$ and $e_i$.
  – O/P:
    • True if you can schedule all the $x_n$ Obeying all the rules.
    • False if not.
Scheduling problem

• An example for the problem is given below.

• I/O

<table>
<thead>
<tr>
<th></th>
<th>S_i</th>
<th>d_i</th>
<th>E_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>3</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>x2</td>
<td>4</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>x3</td>
<td>6</td>
<td>7</td>
<td>20</td>
</tr>
</tbody>
</table>

• O/P

– Yes
To Prove that scheduling problem is NP complete

• We prove that Scheduling problem is NP complete by reduction.

• Subset Sum $\leq_p$ Scheduling Problem
To Prove that scheduling problem is NP complete

- **Subset Sum**
  - **I/P** $S = \{x_1, x_2, x_3, x_4, \ldots, x_n\}, W$
  - **O/P**
    - True if there exists a subset whose sum is $W$.
    - False Otherwise.

- **Each input in S has to be converted to a job.**
  - To convert a number $x_i$ to a job $x_i$
    - $x_i = x_i$
    - $s_i = 0$
    - $d_i = x_i$
    - $e_i = X + 1$, where $X = \sum x_i$
To Prove that scheduling problem is NP complete

• An extra unit job (xn+1) is introduced into it to be scheduled at W. This forces some jobs to be scheduled before W and end at W.
  • \( s_{i+1} = W \)
  • \( d_{i+1} = 1 \)
  • \( e_{i+1} = W + 1 \)

• If there exists a schedule which can satisfies this, then there exists a subset which sums to W.

• Thus this proves that scheduling problem is NP problem.
2 Dimensional Matching

- 2 dimensional matching or Bipartitie matching is a problem in P-Class.
- Consider the diagram below where the left nodes are joined to the right nodes. The problem defines to connect each node on the left to one on the right and get maximum connections.
• This can be solved in polynomial time using max flow algorithm
• One possible solution is given below.
3 Dimensional Matching

- In a 3 dimensional matching we have 3 groups and triples.
- The problem is to find the maximum triples which are disjoint.
3 Dimensional Matching

- We prove that 3 dimensional Matching is a NP complete problem by the following reduction.
- $3 \text{ SAT } \leq_p 3 \text{ DM}$