CSE 6311
Advanced Computational Models and Algorithms

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Lecture notes for March 5, 2009

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OUTLINE

- Euler Circuit
- PTAS for TSP
- Set Cover Approximation

EULER CIRCUIT IN A SIMPLE/MULTI GRAPH

*Euler circuit does not exist for a graph with one or more odd degree nodes

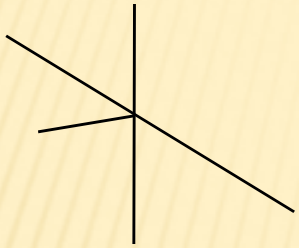


Fig-1

*Euler Circuit Exists for a graph only if every node is of even degree

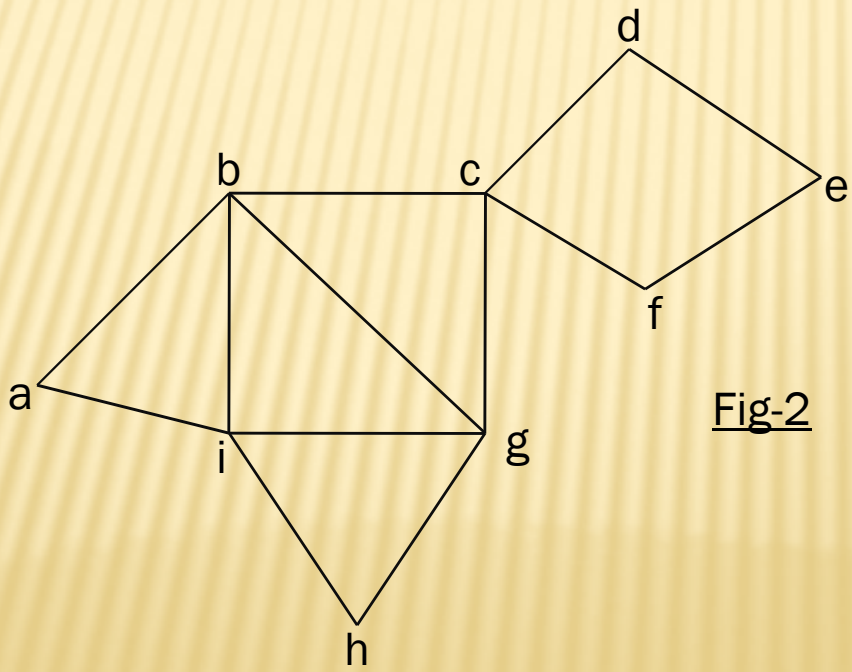


Fig-2

FLEURYS ALGORITHM

Finding Euler Circuit in a graph of even degree at every node.

1.Pick any vertex as a starting point.

2. Marking your path as you move from vertex to vertex, travel along any edges you wish

3. If you find a cycle, separate the vertices in this cycle

4. Find all cycles in graph

5. Continue until you return to your starting point.

6. Patch the cycles

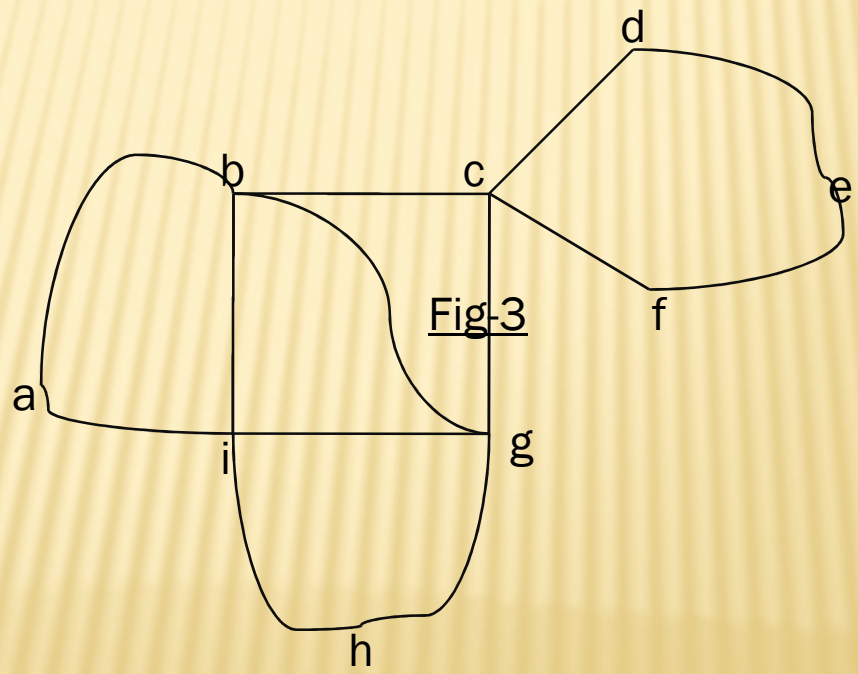


Fig-3

For a traversal of graph in Fig-2 along path **a,b,c,d,e,f,c,g,b,i,h,g,i a** we find the above cycles

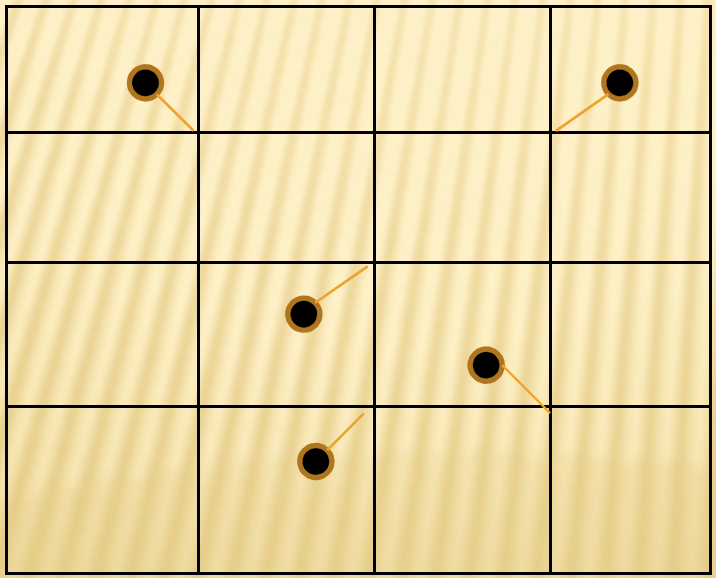
PTAS FOR TSP

Polynomial Time Approximation Schemes break through the hard barrier in Approximation ratios. These scheme design solutions to a NP complete problem as close as we like it to the optimal solution but the tradeoff is the running time.

For a given constant value "Є" $\longrightarrow$ $C_{approx} \leq (1 + Є)\, C_{opt}$

For a TSP problem, If we assume Distances are Geometric/Eucledian/Vector based, we can achieve better Poly. Time Approx. ratios lower than **3/2** ratio(Christofide's Algo)

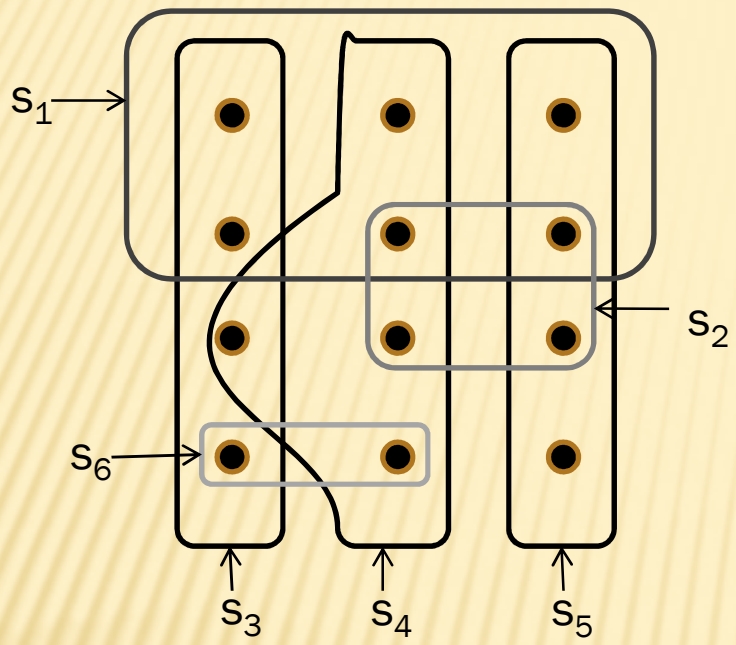
Running Time for TSP = $O(n \log n^{O(1/\varepsilon)})$ *Sanjeev Arora: http://www.cs.princeton.edu/~arora/*



"The idea is to snap the nodes(greater than grid points) onto nearby grid points and apply algorithm on this grid problem"

For a given value of Є = 0.001, It is still in order of $n(\log n)^{1000}$

SET COVER APPROXIMATION



$X = \{1,2,3,.....n\}$

$F = \{ S_1, S_2, S_3, S_4, S_5, S_6 \}$

$S_{opt} \subseteq S$ $\quad |S_{opt}|$ is minimized and

All elements in X are "touched"

The Vertex Cover Approx Algorithm will not work because each edge can touch 2 vertices Only, but in a set each element can belong to many subsets

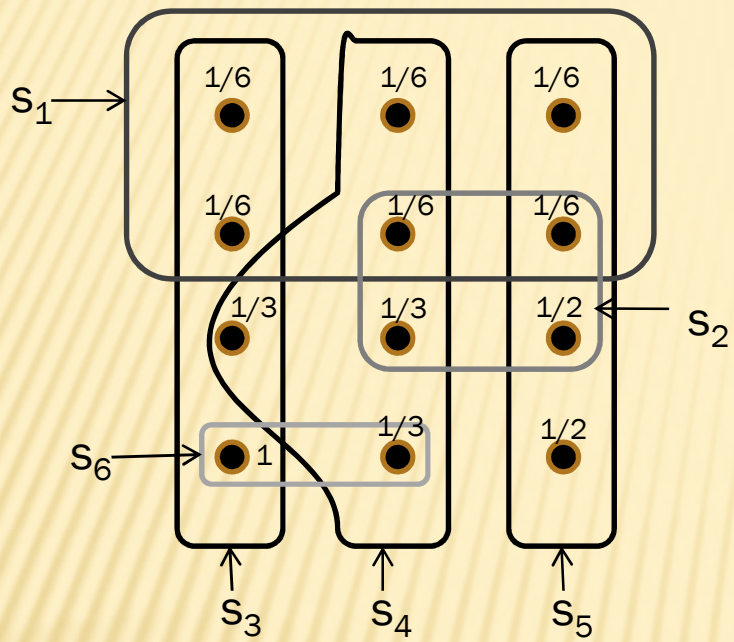
$S_{opt} = \{ S_3, S_4, S_5 \}$

The greedy method works by picking, at each stage, the set S that covers the greatest number of remaining elements that are uncovered.

$S_{approx} = \{ S_1, S_4, S_5 , S_6 / S_3 \}$

We need to prove that the approximation factor is always "**log(n)** "
(where "n" is the number of elements in the set)

AMORTIZED ANALYSIS



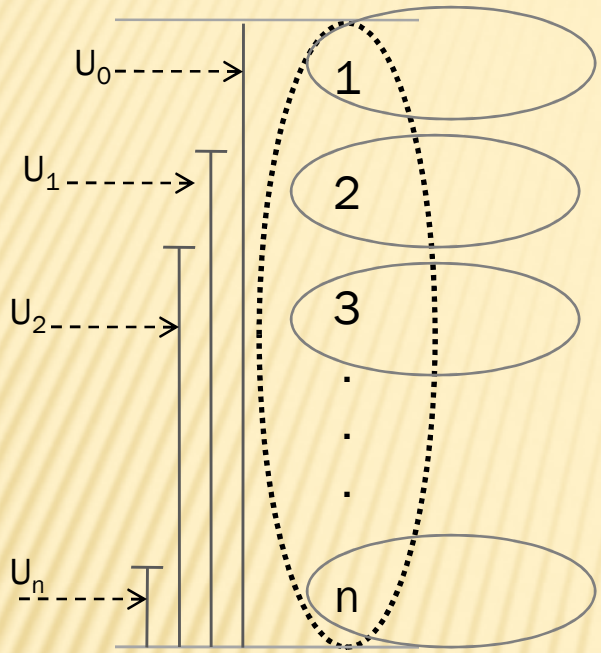
X = { 1, 2, ... i,.....n }

X_i [Mass associated with i^{th} Element]

$$\sum_{i=1}^{n} x_i = C_{approx}$$

$$\sum_{s \in S_{opt}} \sum_{x \in s} x_i \geq C_{approx}$$

$$C_{opt} * \left[\max_{s \in S} \sum_{x_i \in s} x_i \right] \geq C_{approx}$$



i^{th} iteration the number of elements consumed = $U_{i-1} - U_i$

Mass per item $\leq \dfrac{1}{u_{i-1}}$

"The intution is if this set itself was the maximum set then our greedy algo would have chosen it"

$$C_{opt} * \sum_{i=1}^{C_{approx}} (u_{i-1} - u_i) * \frac{1}{u_{i-1}} \geq C_{approx}$$

$$C_{opt} * \sum_{i=1}^{n} \frac{1}{i} \geq C_{approx}$$

$$C_{opt} * \int_{i}^{n} \frac{1}{x} dx = n \ln(n) = nO(\log n) \geq C_{approx}$$

So the ratio is always order of log(n)