CSE 6311
Advanced Computational Models and Algorithms

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CSE UTA
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Outline

- Randomized Algorithm for Quick Sort
- Las Vegas Alg. & Monte Carlo Alg.

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- Analysis of Randomized Algorithm

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Randomized Algorithm for Quick Sort

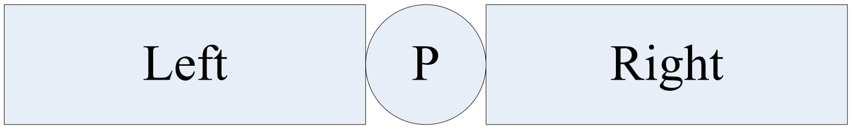
Quick Sort :

- Input: S={x1, x2, …, xn}
- Output: A sorted list of numbers

Quick Sort (S):

1) **Randomly** pick a pivot element P from S
2) Partition S into two parts, elements on the left part are not larger than P and those on the right part are not smaller than P.

$$L \le P \le R$$



3) Quick Sort (Left)
4) Quick Sort (Right)

Randomized Algorithm for Quick Sort

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Analysis of Randomized Algorithm

Steps:

1) The relationship of Xij and T(n)

   Xij is a boolean variable, indicating whether I was compared with j

   (Xij=1, if i is compared with j, otherwise, Xij=0.)

2) The relationship of E [T(n)] and E [Xij]

3) The relationship of Pij and E[Xij]

   Pij is the probability that i is compared with j.

4) Calculate E[T(n)]

Analysis of Randomized Algorithm

Step 1: $$T(n) = \sum_{i=1}^{n-1} \sum_{j>i}^{n} X_{ij}$$

Step 2: $$E[T(n)] = E[\sum_{i=1}^{n-1} \sum_{j>i}^{n} X_{ij}] = \sum_{i=1}^{n-1} \sum_{j>i}^{n} E[X_{ij}]$$

	$X_{1,2}$	$X_{1,3}$	$X_{1,4}$	$X_{1,5}$	…	$X_{n-1,n}$	$\sum_i \sum_j X_{ij}$
1st round	1	0	0	1		1	84
2nd round	1	1	0	0		0	79
.							
.							
.							
infinite							
rounds							
.							
.							
Average X_{ij}= $(\sum_{(round)} X_{ij})$/#of rounds	0.76	0.68	0.9	0.7		0.54	E[T(n)]

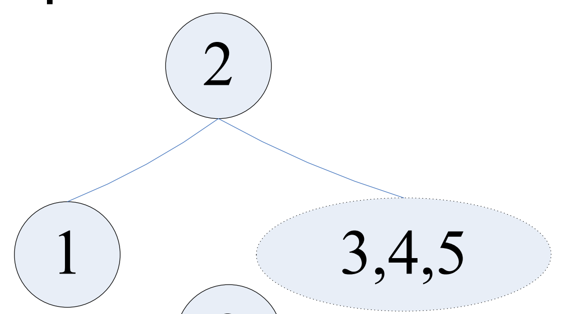
Analysis of Randomized Algorithm

Step 3: $$P_{ij} = E[X_{ij}]$$

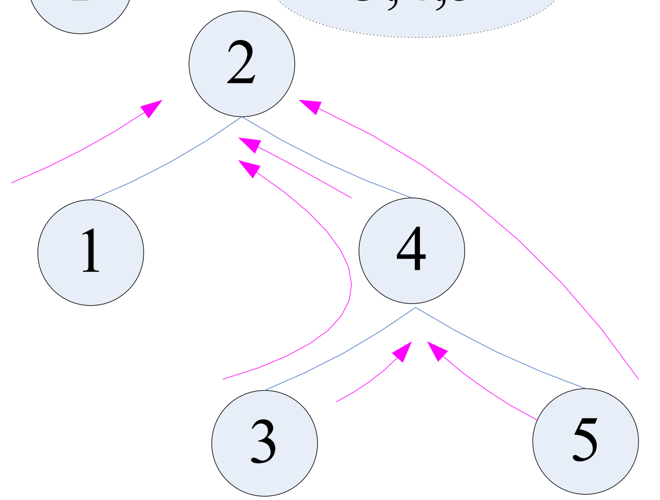
Through the example below, we know that I and j are compared only if one ends up as a descendent of the other.

eg. {1, 2, 3, 4, 5}

First, pick 2



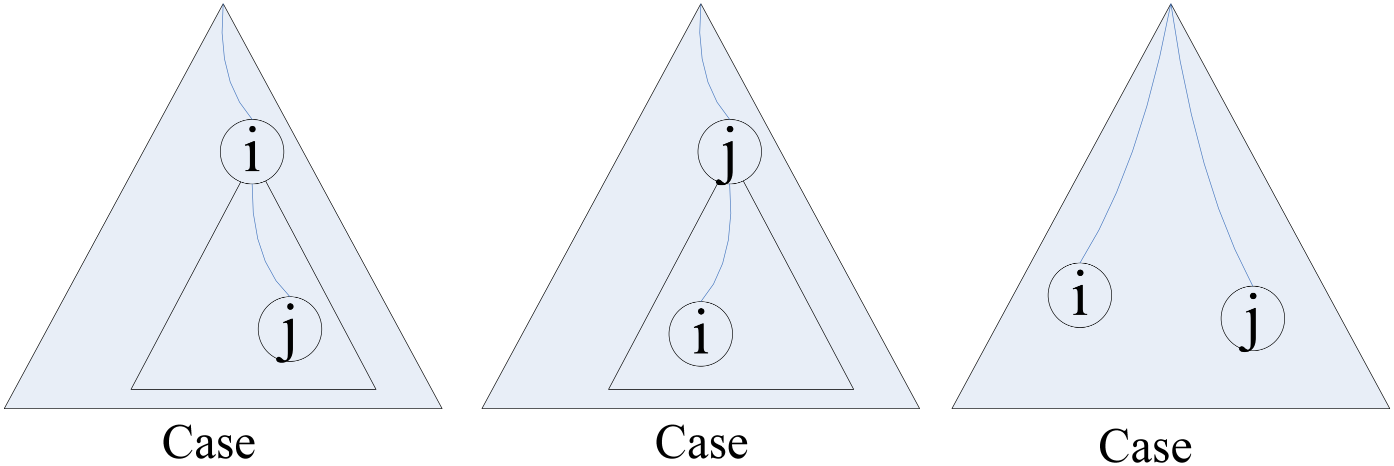
Second, pick 4



Analysis of Randomized Algorithm

Step3:

So for i and j, there are three possible cases:

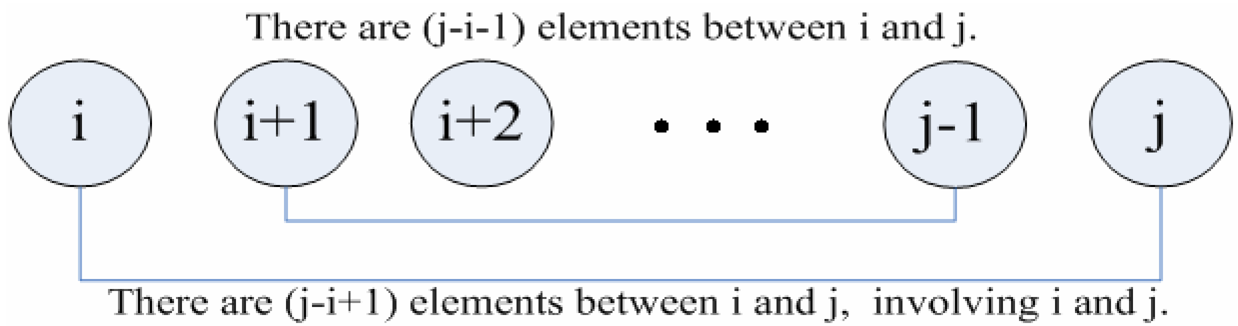


For Case 1 and Case 2, i and j will be compared.

For Case 3, they will not be compared.

Analysis of Randomized Algorithm

Step3:



If we select elements between i+1 and j-1, i and j will never be compared.

If we select elements i or j, i and j will be compared.

Conclusion:

$$P_{ij} = \frac{2}{j - i + 1}$$

Analysis of Randomized Algorithm

Step 4:
$$E[T(n)] = \sum_{i=1}^{n-1} \sum_{j>i}^{n} \frac{2}{j-i+1} = \sum_{i=1}^{n-1} \sum_{k=2}^{n-i+1} \frac{2}{k}$$

$$\leq \sum_{i=1}^{n} \sum_{k=1}^{n} \frac{2}{k} = 2n \sum_{k=1}^{n} \frac{1}{k} = O(n \lg n)$$

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Las Vegas Alg. & Monte Carlo Alg.

Las Vegas Alg.

1) It always produces the **right** answer.

2) Its running time is a **random variable.**

Monte Carlo Alg.

1) The answer is **approximate.**

2) Its running time is bounded by a **fixed approximated/deterministic** value.

Las Vegas Alg. & Monte Carlo Alg.

Min Cut Graph:

Find the **smallest** set of edges so that removing them will make the graph disconnected.

For **deterministic algorithm**, for each pair of nodes, run Maximum Flow Algorithm to find the min cut and select the smallest set.

For **randomized algorithm**, assume the size of the opt min cut is k, if the graph has n vertices, the number of edges is not smaller than nk/2, since the degree of each node is at least k.

Las Vegas Alg. & Monte Carlo Alg.

The min cut in the graph below is 2.