ADVANCED COMPUTATIONAL MODELS AND ALGORITHMS

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Lecture 9

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Outline

- Maximum Independent set Problem
- Traveling Salesmen problem
- Metric Traveling salesmen problem

Maximum Independent Set problem

Even though two problems are related like Vertex cover and Max-Independent set problem,having an approximation algorithm for one does not mean that we have one for another too.
Now we are going to prove this fact.

G=(V,E) an undirected graph
|V|=n total number of vertices in the graph
|E|=m total number of edges

let Vcover be the vertex cover of G
Vcover <=V
V-Vcover is a corresponding independent set

Vaprox-> Approximate vertex cover.
Vopt----> Optimal vertex cover
Iaprox--> Approximate Independent set
Iopt ----> optimal Independent set

Vaprox <= 2Vopt -------1
ie the vertex cover got by aprosimation algorithm is at max 2 times the optimal solution of vertex cover

V-Vaprox = Iapprox----2
Iopt= V-Vopt -----3

Now to express Iaprox as a factor of Iopt as in the equation 2 for vertex cover, we need a constant factor say 'C'.
Iaprox>= C Iopt

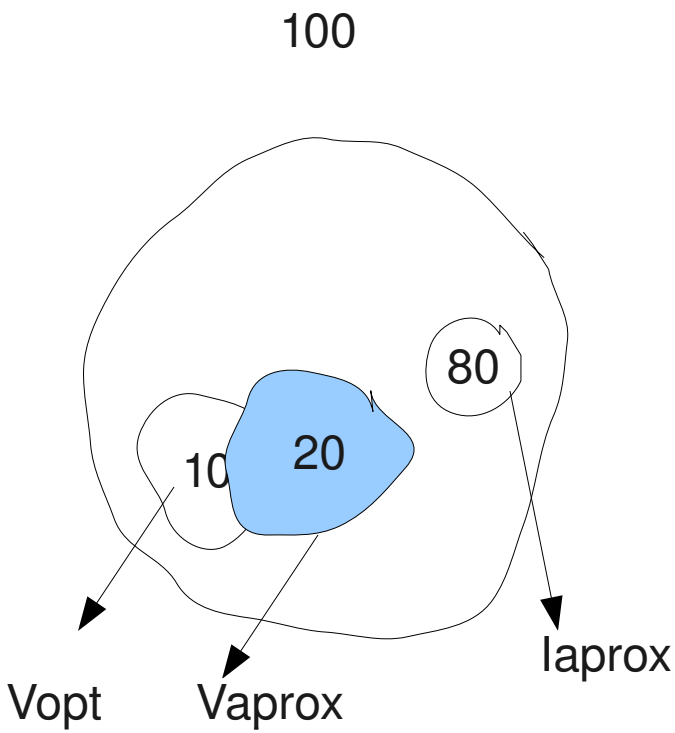
we are interested in finding the constant C

let |Vopt|=x
|Vaprox|=y
|Iaprox|= z
|Iopt|=u

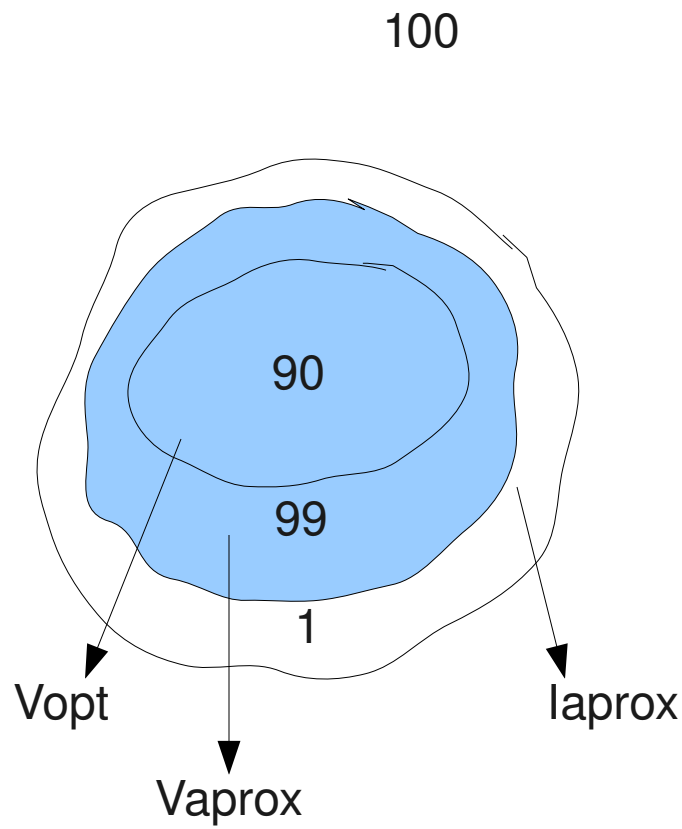
Now from the equations 1,2 & 3 we get
y<=2x-----4
n-y=z------5
u=n-x------6
and we know that x<=y<=n

we need the relation between z and u
from the equation 4,5and 6 we get
y=n-z
x=n-u
y/x=n-z/n-u<=2
n-z<=2n-2u
z>=2u-n
it is impossible to deduce this equation further to express u as a factor of z.

Considering a graph of 100 vertex, the fact that the relationship cannot be established between a vertex cover approximation algorithm and Max Independent set is shown below



V-Vopt=90=Iopt is close to approx
so the Iaprox here is acceptable



V-Vopt=10=Iopt is a not near the Iaprox
ie. Iopt is a factor of 10 with respect to Iaprox
which is not acceptable

So no particular constant exists to connect Iaprox and Iopt and thus no relation exists.

Traveling Salesman Problem

Input:

G=(V,E) a weighted complete graph

Output:

The shortest tour which visits each vertex exactly once.

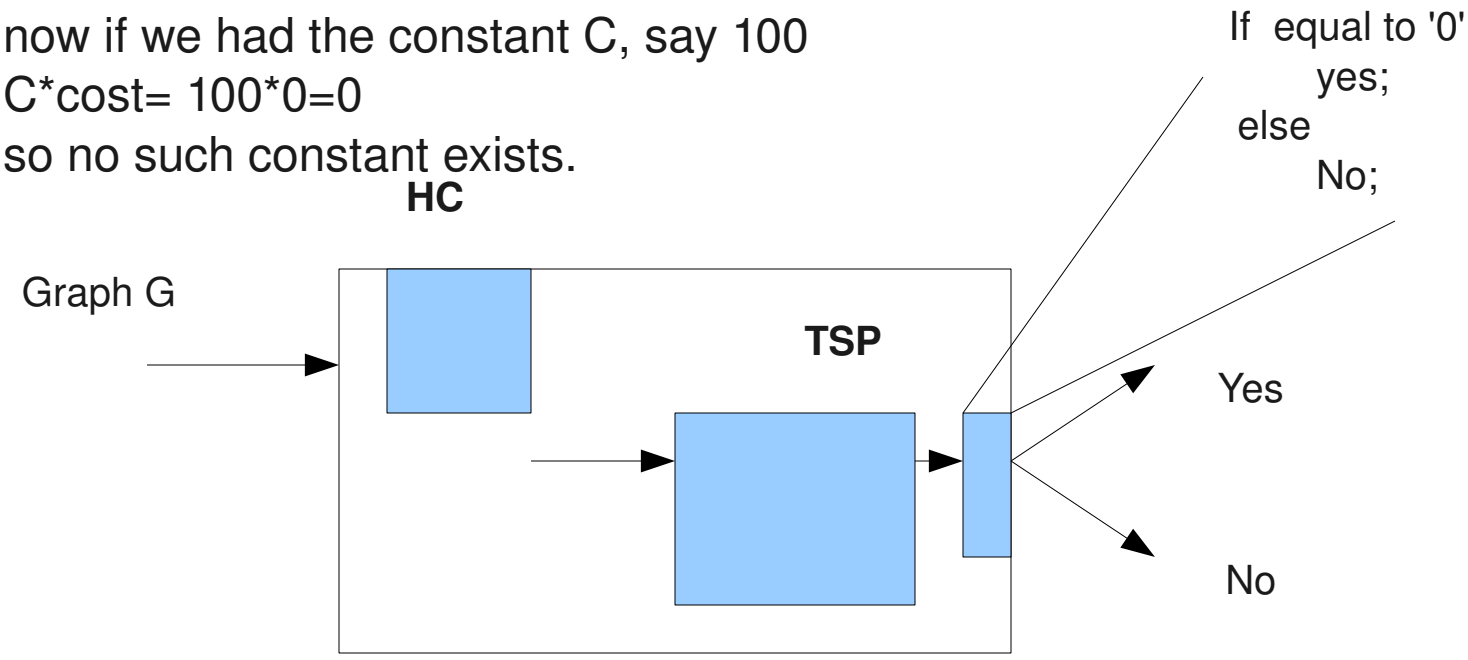
For the general problems, a constant factor approximation is impossible unless P=NP

Proof:

Assume ∃ optimization algorithm 'A' that run in polynomial time grantees a approximation factor of c>=1

now if we have an algorithm to solve the TSP problem
 for a given graph G , we add all the remaining edges to make the graph complete and give a weight of '0' to the original edges and '1' to the newly added edges.
And we ask the TSP solution black box whether a tour exists of cost 0.

Now the TSP solution black box finds a circuit of cost '0'
now if we had the constant C, say 100
C*cost= 100*0=0
so no such constant exists.



General TSP cannot be approximated by a general factor

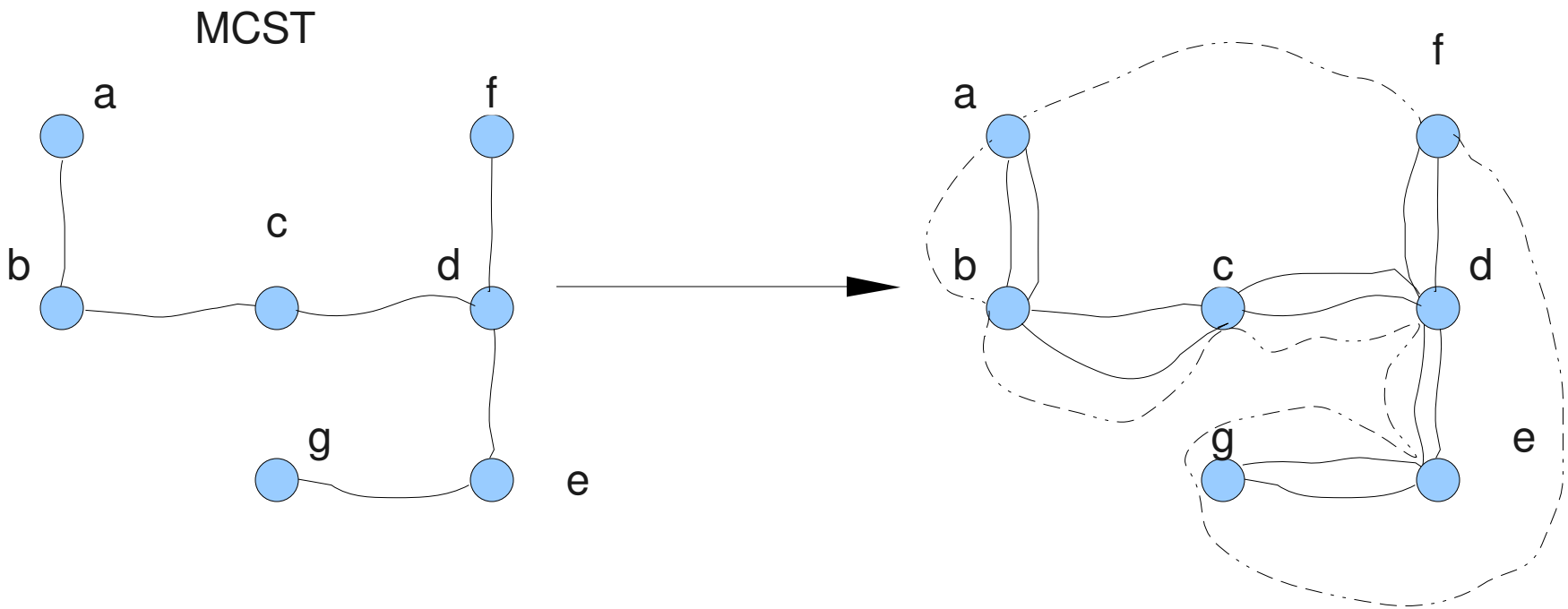
Metric TSP Problem

This is a restriction version of TSP.
A very natural restriction of the TSP is to require that the distances between cities form a metric, i.e., they satisfy the triangle inequality. That is, for any 3 cities A, B and C, the distance between A and C must be at most the distance from A to B plus the distance from B to C. Most natural instances of TSP satisfy this constraint.

A simple greedy algorithm for this is, for each step chose the shortest distance of the remaining edges.

An algorithm which is efficient than this is discussed in the next page

We construct a Minimum spanning tree from the given graph, and add an extra edge between each vertex between which an edge already exists.



After adding an extra edge we go through each vertex and tour back until all the vertex are covered but while coming back we consider only the unvisited edges and draw an edge between last visited and the current unvisited vertex (doted line) which gives us the second graph above an TSP

Topt >= MST (viewing TSP as one more edge to MCST

Tprime=2 MCST (we can see this in the graph last page)

Tprime>=Tapprox

Taprox>= 2 Topt

Note: a factor of 3/2 approximation algorithm also available

we can do this using Maximum weight machine in graph