Class notes for CSE 6311 Adv. Comp. Models and Algorithms

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Outline

1. Approximation Algorithms
    - Fully Polynomial-time Approximation Scheme for Subset Sum Problem
    - Constant-bound Polynomial-time Approximation Algorithm for Subset Sum Problem

Outline

Fully Polynomial-time Approximation Scheme for SSP

Overview

- Exact algorithm based on dynamic programming features $O(nW)$ time complexity
- Fully PTAS consists in compressing the list of partial solutions at each iteration by storing only solution y_i to represent all solutions $y_j \in [y_i, (1+\delta)y_i]$ where $\delta = \varepsilon/2n$
- At the last iteration in L_n, $y_{opt} \leq (1+\delta)y_m$ where y_m is the last representing solution chosen by the approximate algorithm
- Thus, $y_{approx} \leq y_{opt} \leq (1+\delta)^n y_{approx}$ as the approximate solution is compressed at most n times

Fully Polynomial-time Approximation Scheme for SSP

Proof of Approximation Scheme

- In order to prove that this an approximation scheme driven by ε, we need to prove that
  $(1+\delta)^n \leq 1+\varepsilon$
- Proof:
  $(1+\delta)^n = (1+\frac{\varepsilon}{2n})^n = 1+n(\frac{\varepsilon}{2n}) + \binom{n}{2}(\frac{\varepsilon}{2n})^2 \ldots \leq 1+\varepsilon$
- Thus, $y_{approx} \leq y_{opt} \leq (1+\varepsilon)y_{approx}$

Fully Polynomial-time Approximation Scheme for SSP

Proof of Polynomial-time Complexity

- In the worst-case intervals cover from the first element of the list y_1 through W
  $y_1(1+\delta)^m = W$
  $m = \log_{1+\delta} \frac{W}{y_1} \leq \log_{1+\delta} W = \log_{1+\frac{\varepsilon}{2n}} W = \frac{\ln W}{\ln(1+\frac{\varepsilon}{2n})} \leq \frac{\ln W(1+\frac{\varepsilon}{2n})}{\frac{\varepsilon}{2n}} = \frac{2n(1+\frac{\varepsilon}{2n})\ln W}{\varepsilon}$
- Thus, $m = O(n, \ln W, \frac{1}{\varepsilon})$, which is characteristic of a fully polynomial approximation scheme

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Constant-bound Polynomial-time Approximation Algorithm for SSP

Proof that it doesn't exist unless P = NP

- Polynomial-time algorithm s. t. $\mid y_{opt} - y_{approx} \mid$ does not exist for SSP unless P = NP
- Proof by contradiction
  - Assume such an algorithm A with $c \neq 0$ exists
  - Given an instance of SSP: $S = s_1, s_2, \ldots s_n, W$
  - Create an instance of SSP:
    $S = (c+1)s_1, (c+1)s_2, \ldots (c+1)s_n, (c+1)W$
  - By definition $S'_{opt} = (c+1)S_{opt}$, and $\mid S'_{opt} - S'_{approx} \mid \leq c$
  - Thus, $\mid S_{opt} - S_{approx} \mid \leq \frac{c}{c+1} = 0$ i. e., polynomial-time algorithm A yields optimal solution for NP-complete problem SSP