# CSE 6311

# Advanced Computational Models and Algorithms

Instructor: Dr Gautam Das

Lecture notes
for
January 22, 2009

Compiled by Giacomo Ghidini

# Outline

- Polynomial time computability
- Reductions
- Optimization, decision and verification versions of computational problems
- Classes P, NP

# Polynomial time computability

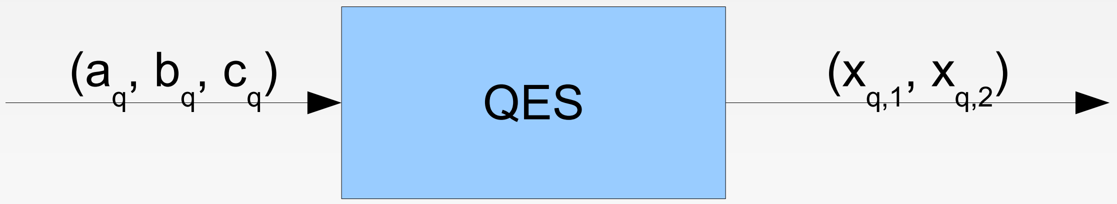
- Class of polynomial functions
  - $f(x) = O(x^k)$
  - Closed over product
    - E. g., product of polynomials is also a polynomial
    - This does not apply to the class of cubic functions

# Polynomial time computability

- Problem
  - Solving a linear equation
    - E. g., given a, b, find the value of x s. t. $ax + b = 0$
- Solver
  - Given the linear equation problem (LEP), a linear equation solver (LES) is defined as a black box with input $(a_I, b_I)$ and output $x_I$

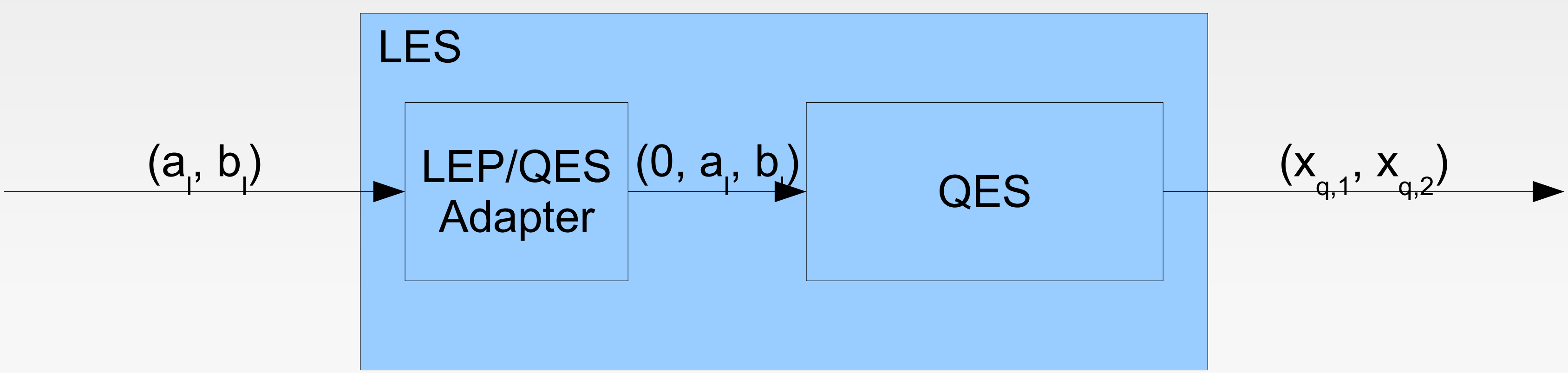
# Polynomial time computability

- Problem
  - Solving a quadratic equation
    - E. g., given a, b, c, find the value of x s. t. $ax^2 + bx + c = 0$
- Solver
  - Given the quadratic equation problem (QEP), a quadratic equation solver (QES) is defined as a black box with input $(a_q, b_q, c_q)$ and output $(x_{q,1}, x_{q,2})$



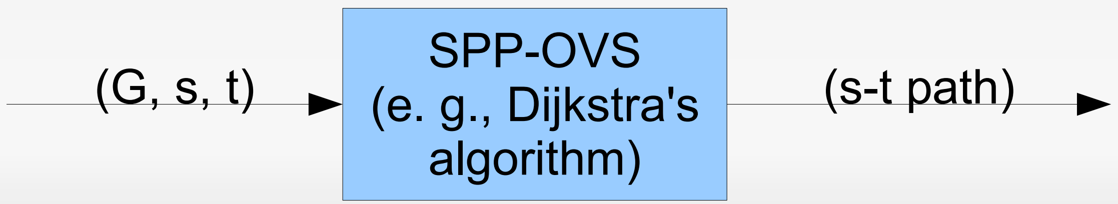
# Reduction

- Problem
  - Solving a linear equation
- Solver
  - A black box that reduces a LEP to a QEP and then uses a QES to solve the original LEP



# Optimization and decision versions

- Shortest Path Problem-Optimization Version
  - Given a graph G = (V, E) and two vertices s and t, find the minimum cost path between s and t
- Solver
  - Given the shortest path problem optimization version (SPP-OV), a SPP-OV solver (SPP-OVS) is defined as a black box with input (G, s, t) and output (s-t path)

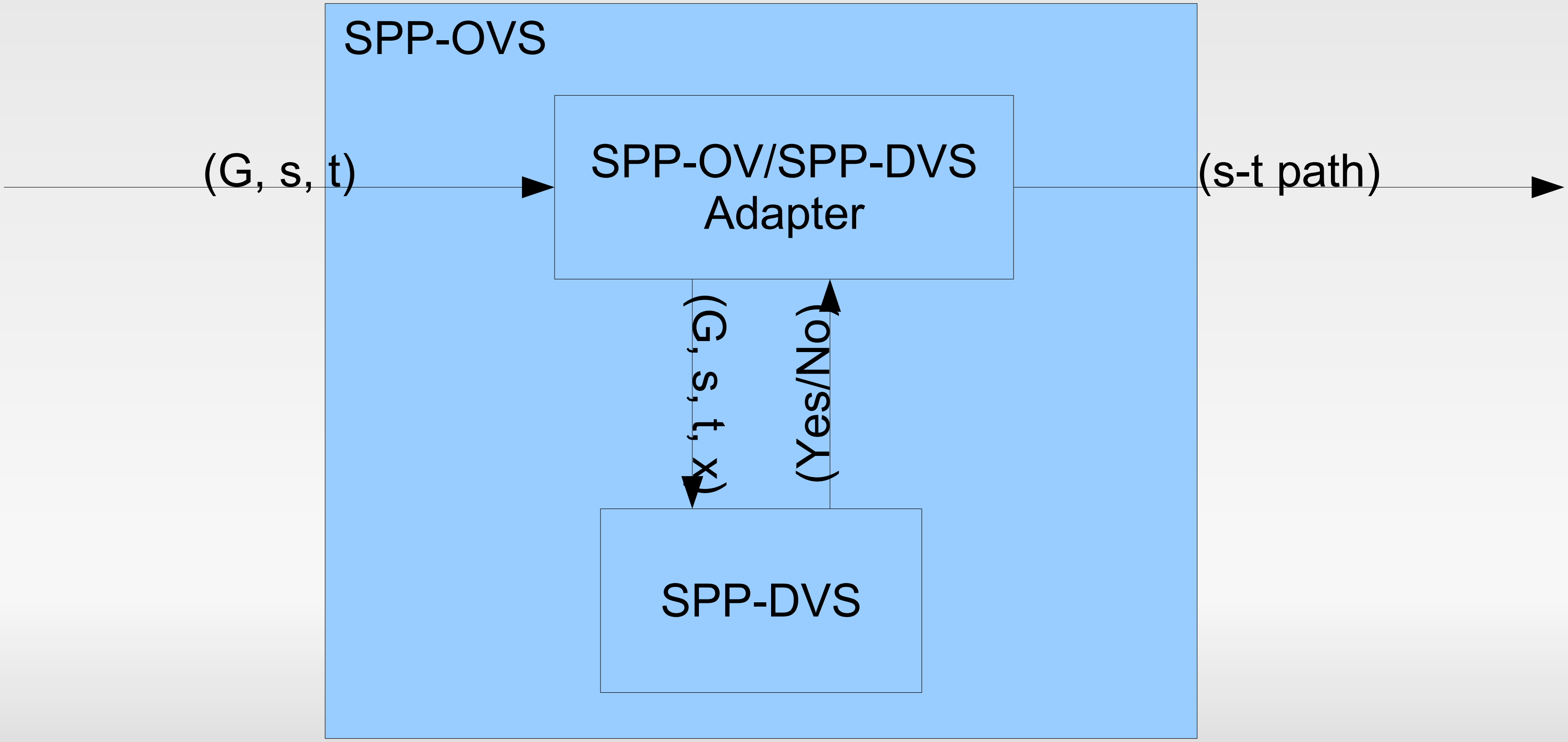


# Optimization and decision versions

- Shortest Path Problem-Decision Version
  - Given a graph G = (V, E), two vertices s and t and a cost x, say whether there exists an s-t path of cost ≤ x
- Solver
  - Given the shortest path problem decision version (SPP-DV), an SPP-DV solver (SPP-DVS) is defined as a black box with input (G, s, t, x) and output (Yes/No)

# Reductions between OV and DV

- Solving SPP-DV with SPP-OVS is feasible
- Solving SPP-OV with SPP-DVS looks more complicated but it's also feasible



# Verification version

- Shortest Path Problem-Verification Version
  - Given a graph G = (V, E), two vertices s and t, a cost x and an s-t path (certificate), say whether the s-t path is of cost ≤ x
- Solver
  - Given the shortest path problem verification version (SPP-VV), an SPP-VV solver (SPP-VVS) is defined as a black box with input (G, s, t, x, s-t path) and output (Yes/No)

# Complexity of optimization, decision and verification versions

- Optimization and decision versions are equally difficult
  - Pay attention in developing an OV/DVS adapter such that it has polynomial time complexity and thus its composition with a polynomial time algorithm is still polynomial
- Verification version is usually easy, this being independent of OV and DV complexity
  - E. g., Longest Path Problem OV and DV haven't polynomial time algos, while LPP VV is clearly polynomial time
  - This applies to NP problems

# Classes P and NP

- Class P
  - The set/class of decision version problems that can be solved in polynomial time
- Class NP
  - The set/class of verification version problems that can be solved in polynomial time
- $P \subseteq NP$ easy to observe from definitions
  - Turing machine is needed for formal proof
- $P = NP$ is an open issue