ADVANCED COMPUTATIONAL MODELS AND ALGORITHMS

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Outline

- Subset Sum – Decimal
- Subset Sum – Binary
- Subset Sum – Unary

Subset Sum (Decimal)

Decision Version :

Input :

S - set of decimal integers

t - an integer

Output :

Yes , if there is a subset $S' \subseteq S$ s.t. $\sum x = t$, where $x \in S$

No, otherwise

Subset Sum (Decimal)

Subset Sum is NP Complete :

Step 1 - Subset sum is in NP

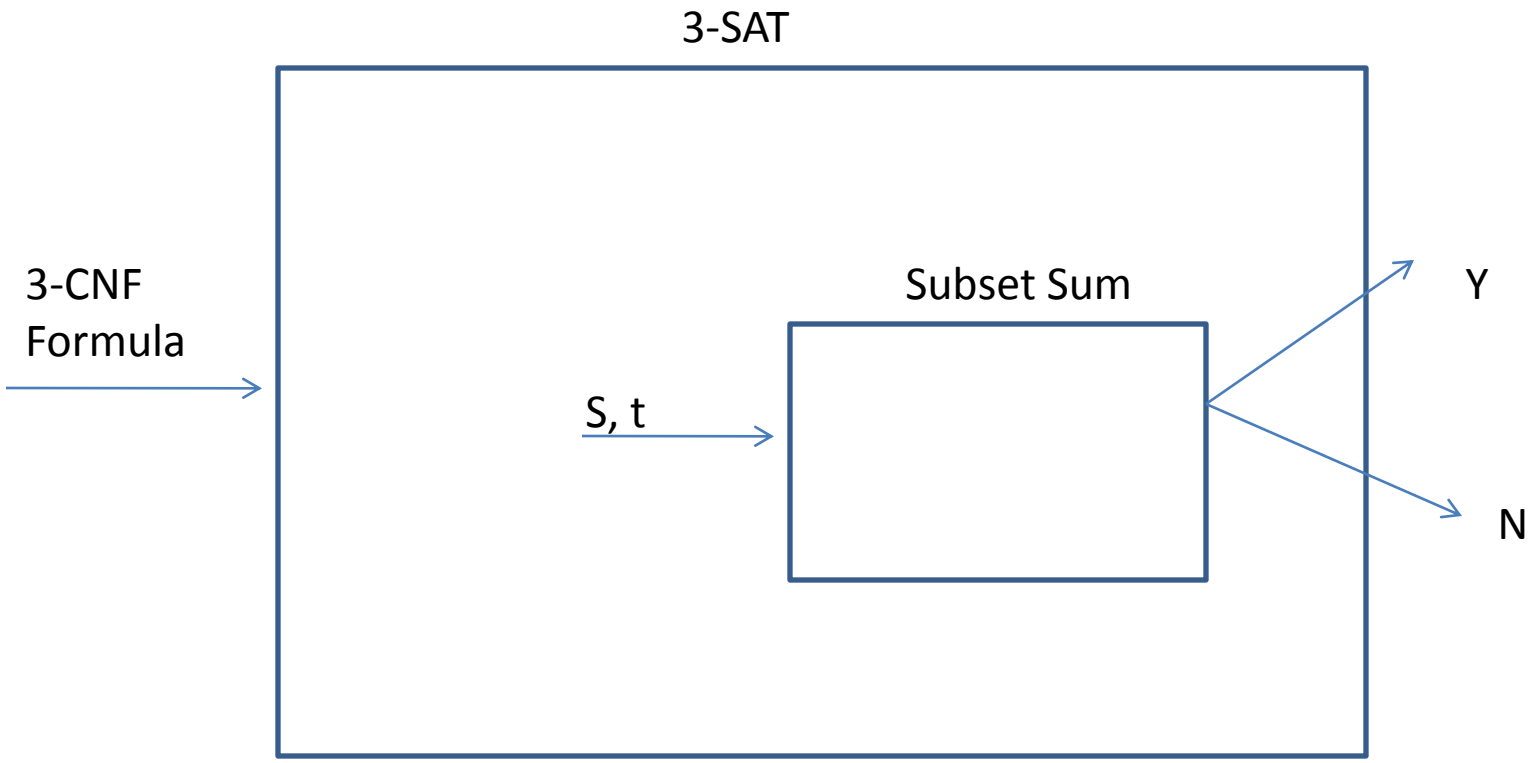
- Certificate is the subset S’
- Verification can be done in polynomial time.

Step 2 - Reduction

- Reduction using 3-CNF

Subset Sum (Decimal)

Reduction :



Reduction (contd):

- Input is 3-CNF formula with ***n*** variables and ***m*** clauses

- Eg : f = c1 ^ c2 ^ c3 ^ c4
    - C1 = (x1 v ¬ x2 v ¬x3)
    - C2 = (¬ x1 v ¬ x2 v ¬x3)
    - C3 = (¬ x1 v ¬ x2 v x3)
    - C4 = (x1 v x2 v x3)

- Here m = 4 and n = 3
- One satisfying assignment is (x1 = 0, x2 = 0, x3 = 1)

Reduction (contd):

Table Construction

- For each variable x , create two numbers (rows) x and x'
- For each clause C_i , create two numbers (rows) C_i and C_i'
- All numbers are in *base 10*
    - This prevents carries during addition.
- Each number has *n+m* digits
- Each digit corresponds to a clause or a variable.
- Leading n digits are labeled by variables
    - A digit x_i has 1 for variables x_i and x_i'
- Trailing m digits are labeled by clauses
    - A digit C_i has 1 for variable x_i , if x_i occurs in C_i

	X1	X2	X3	C1	C2	C3	C4
X1	1	0	0	1	0	0	1
X1’	1	0	0	0	1	1	0
X2	0	1	0	0	0	0	1
X2’	0	1	0	1	1	1	0
X3	0	0	1	0	0	1	1
X3’	0	0	1	1	1	0	0
C1	0	0	0	1	0	0	0
C1’	0	0	0	2	0	0	0
C2	0	0	0	0	1	0	0
C2’	0	0	0	0	2	0	0
C3	0	0	0	0	0	1	0
C3’	0	0	0	0	0	2	0
C4	0	0	0	0	0	0	1
C4’	0	0	0	0	0	0	2
t	1	1	1	4	4	4	4

Notes :

- **t** has **1** in each digit labeled by a variable and **4** in each digit labeled by a clause.
- All the numbers (rows) generated are unique.
- Greatest digit in any position is **6**. (So any base b >= 7 will work for this reduction)
- The table has 2(m+n) rows.
- The rows for clauses (C_i and C_i') act as make-up rows. (to make the sum as 4) –
  - For any clause Ci with 3 variables then in a satisfying assignment ,
    - If one of those variables is true, then both rows Ci and Ci' make up and sum becomes 4
    - If two variables is true , then Ci makes up and sum becomes 4
    - If all three variables is true , then Ci' makes up and sum becomes 4

Notes :

- Transformation from 3-CNF to table is in polynomial time.
- 3 – CNF is satisfiable if and only if there is a subset S’ of S whose sum is t.
- Only one of x_i or x_i’ rows can be included in the subset.

Subset Sum (Binary)

Decision Version :

Input :

S - set of binary integers

t - an integer

Output :

Yes , if there is a subset $S' \subseteq S$ s.t. $\sum x = t$, where $x \in S$

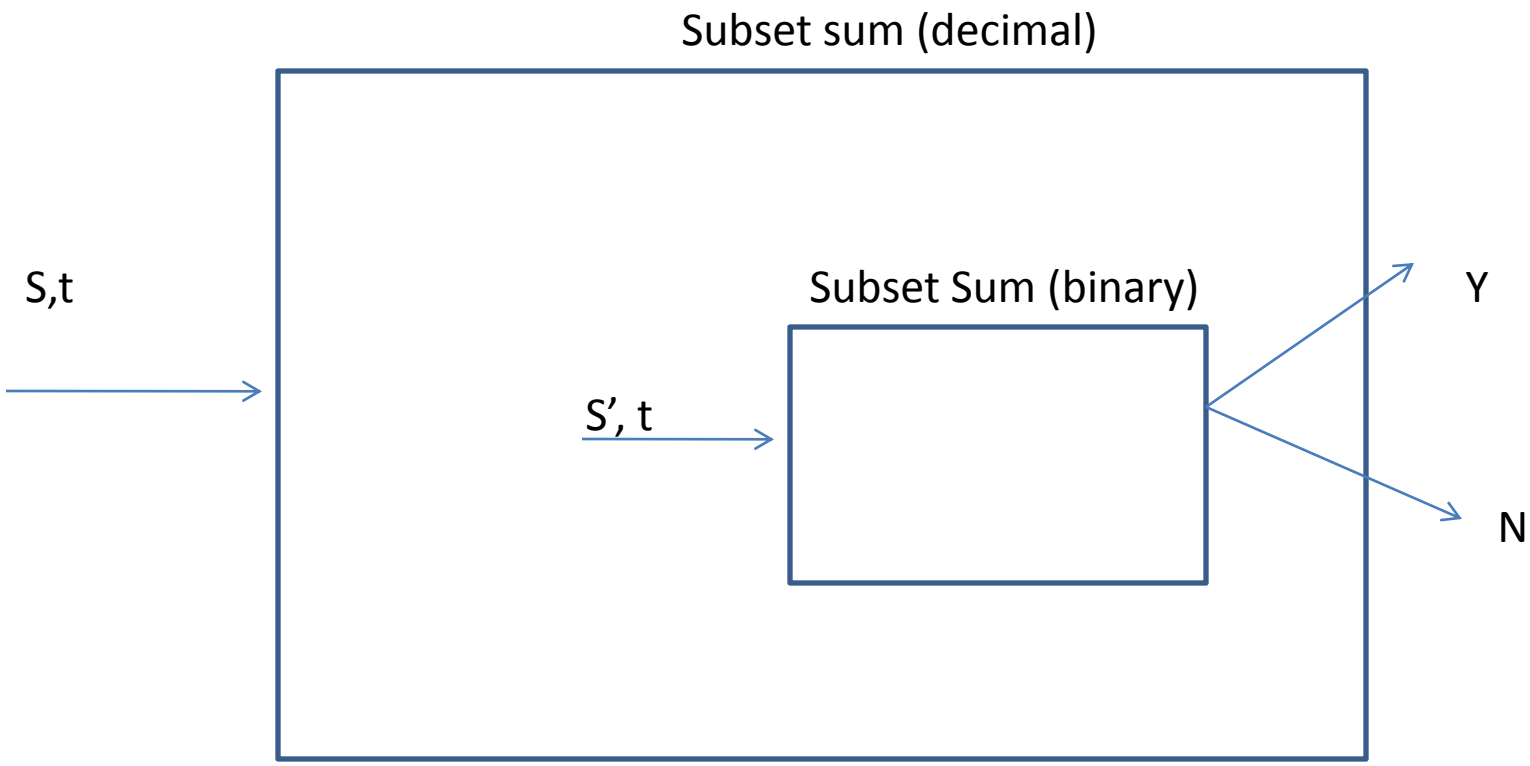
No, otherwise

Notes :

- Use Subset sum (decimal) for reduction.
- Decimal to binary can be converted in polynomial time.
- If a decimal number has k digits, then its binary number has atmost 3k digits.

Subset Sum (Binary)

Reduction :



Subset Sum (Unary)

Unary representation :

1 = 1 , 11 = 2 , 111 = 3 and 1111 = 4

Decision Version :

Same as subset sum (binary) except integers are in unary representation.

- Conversion of an integer (in decimal/binary) to unary takes exponential time.
- Since conversion is not in constant time, we cannot use reduction technique to prove if it is NP-complete.
- Subset sum (Unary) is in P.
  - It can be solved using dynamic programming.
- Subset sum is a weakly NP complete problem as it is in P when encoded in unary and NP-Complete else (base -2,10)