MiniMax Principle in Game Theory

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➢ MiniMax Principle from Game Theory is used to provide a lower bound of the running time of the Randomized AND-OR tree algorithm.

➢Rachael and Chris play a game with three objects:

- Stone
- Paper
- Scissors

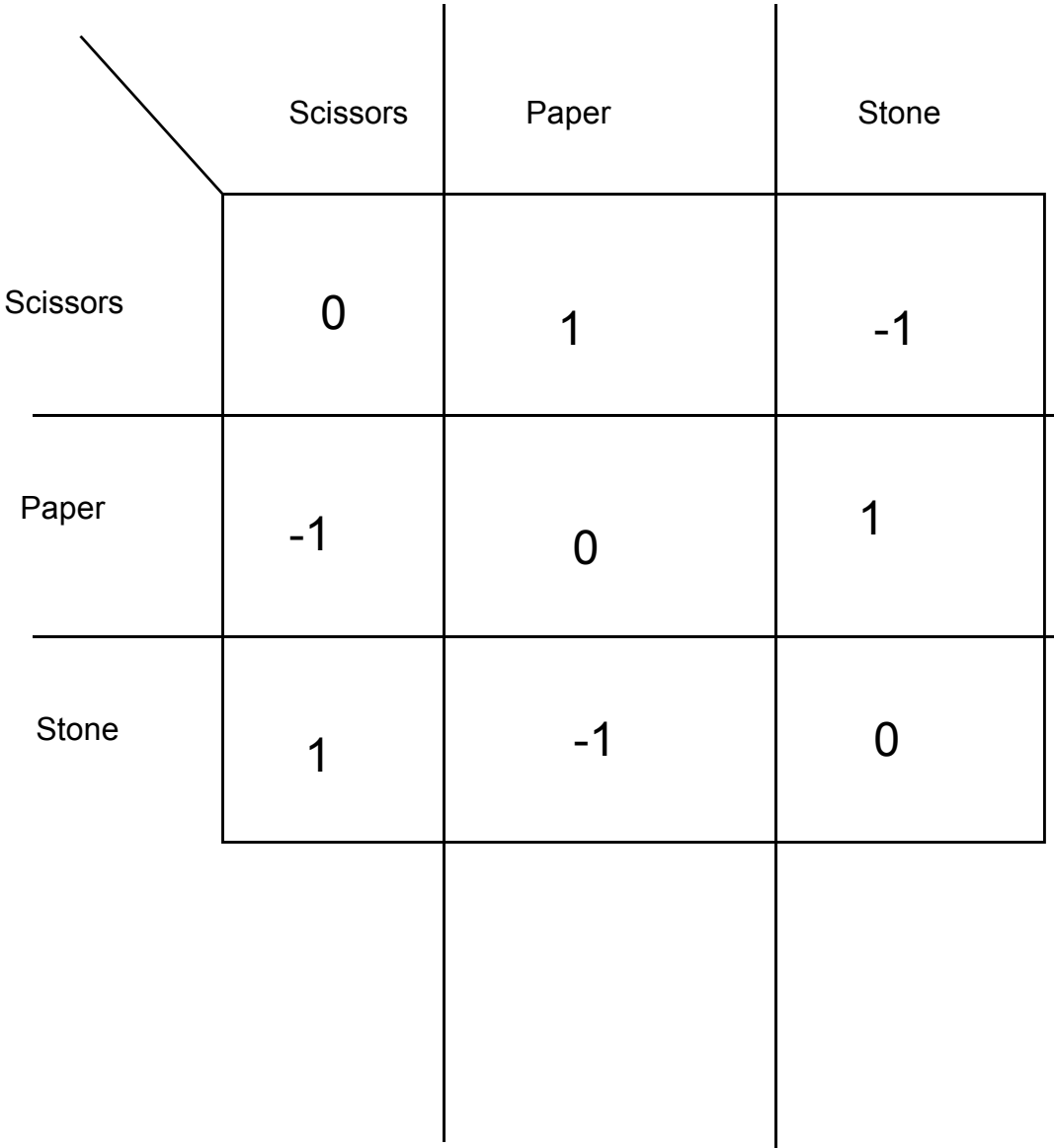
➢ In this simultaneous game,both Rachael and Chris pick up one of these three objects.

➢ This three objects have the following order of preference :

Stone >> (preferred than) Scissors
Scissors >> Paper and
Paper >> Stone.

The winner of the game Played by Rachael and Chris is determined by the following way:

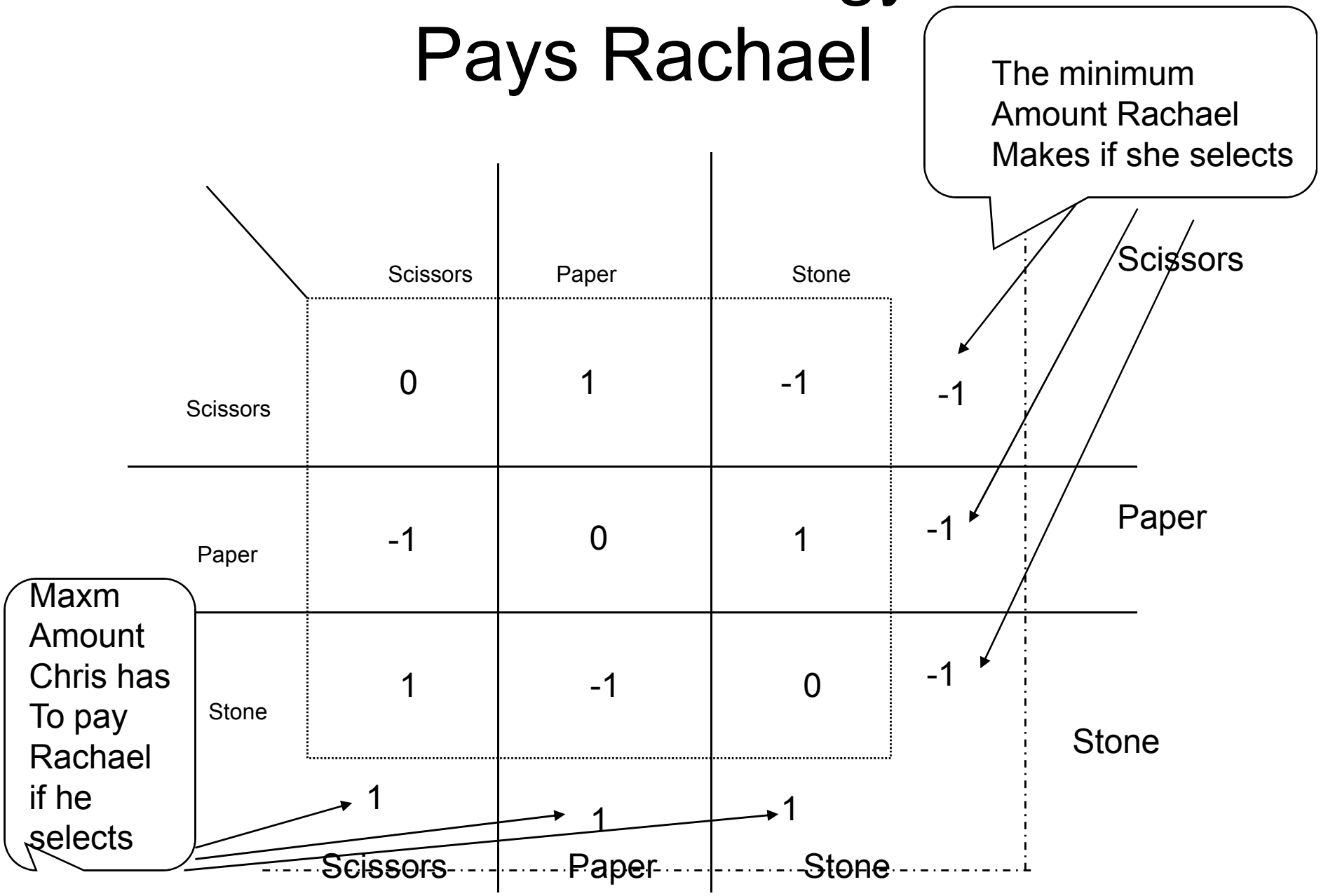
The person who picks up a more preferred item than the other is the winner. The loser pays $1 to the winner and the outcome is a draw when both of them picks up the same object



Matrix for Scissors-Paper-Stone

This is called Two Person Zero Sum Game since Money are Getting exchanged Between hands and There is no External money flow.

A Deterministic Strategy – Chris Pays Rachael



Another Pay Off Matrix

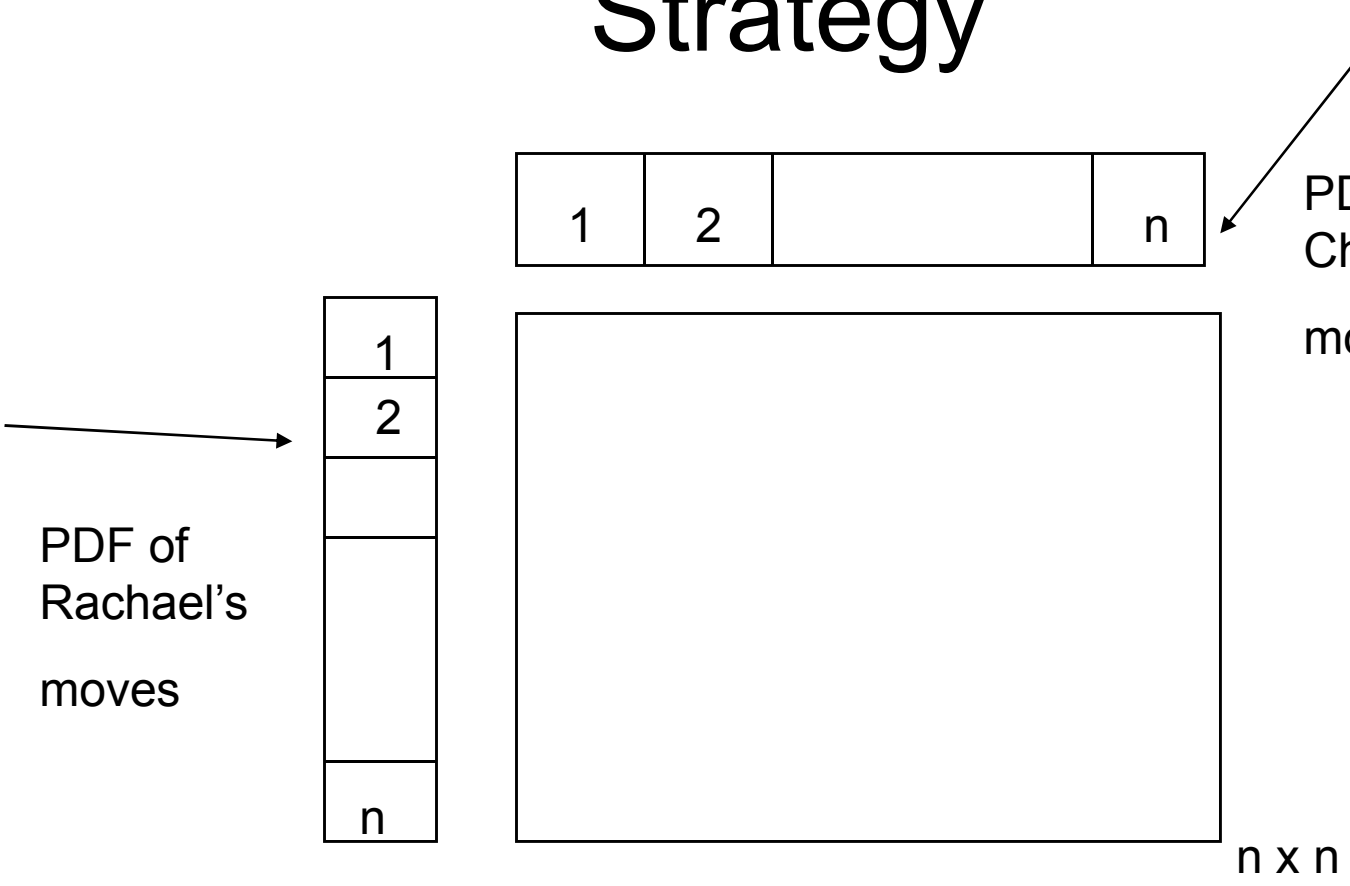
	Scissors	Paper	Stone	
Scissors	0	1	2	0
Paper	-1	0	1	-1
Stone	-2	-1	0	-2
	0	1	2	$V_R = \max_i\{\min_j M_{ij}\}$
	$V_C = \min_j\{\max_i M_{ij}\}$			$V_R = 0$
			$V_C = 0$	

What is V_R and V_c in a Conservative Game Strategy?

- $V_{R =}$ The lower bound of the amount of money that Rachael can make/round
- V_c = The upper bound of the money that Chris can give to Rachael.
- And in general $V_R <= V_c$
- For certain Pay-off matrices ,

$$V_R = V_c$$

Probabilistic Game Playing Strategy



Expected [payoff] = $(\Sigma\ i=1\ ..n)\ (\Sigma\ j=1\ ..n)\ p_i M_{ij} q_j = p^T M q$

How does this strategy work? - Von Neumann's MiniMax Principle

- Say Rachael has selected p. Then Chris should select a distribution q such that $P^T Mq$ is minimized.
- $\text{Min}_q\ P^T Mq$.
- Rachael wants $V_R = \max_p \min_q P^T Mq$.
- Chris wants $V_C = \min_q \max_p P^T Mq$.
- In the probabilistic game playing strategy , $V_R = V_C$
- This is known as Von Neumann's MiniMax Principle.

PDF interpretation of Deterministic Game:

❖All moves get 0 as the probability and the move chosen by the player gets 1.

❖PDF looks like

0	0	0	1	0	- - - - - -	0

Loomi's Theorem:

It says

$\max_p \min_e P^T M e = \min_q \max_f f^T M q$, where e and f are special distributions has only 1 and 0 everywhere else.