The probability that a random variable deviates by a given amount from its expectation is referred to as a tail probability.
It is a problem of assigning \( m \) indistinguishable objects\( (\text{“balls”}) \) to \( n \) distinct classes\( (\text{“bins”}) \).

One ball is thrown at a time with equal probability of landing into any bin.
Probability that no. of balls in bin 3 be i

It can be found out by finding the probability of bin 3 being selected i times.

The probability distribution is a Binomial Distribution. So,

\[ P_i = \binom{n}{i} \left(\frac{1}{n}\right)^i \left(1-\frac{1}{n}\right)^{n-i} \]
**Expected Number** of balls in each bin $E[X]$ is given by $np$, where $n$ is the number of bins and $p=1/n$ is the probability of any bin getting selected at each iteration.

**Standard Deviation** is given by $\sqrt{n \cdot \frac{1}{n} \cdot \frac{n-1}{n}} = \sqrt{\frac{n-1}{n}} < 1$
Probability that a bin has \( k \) or more balls
\[ \sum_{i=k}^{n} nCi \left( \frac{1}{n} \right)^i \left( 1 - \frac{1}{n} \right)^{n-i} \leq \sum_{i=k}^{n} nCi \left( \frac{ne}{i} \right)^i \left( \frac{1}{n} \right)^i \, \text{Since} \ldots nCi \leq \left( \frac{ne}{i} \right)^i \]

\[ \leq \sum_{i=k}^{n} \frac{e^i}{i} \]

\[ = \left( \frac{e}{k} \right)^k + \left( \frac{e}{k+1} \right)^{k+1} + \left( \frac{e}{k+2} \right)^{k+2} + \ldots \]

\[ \leq \left( \frac{e}{k} \right)^k + \left( \frac{e}{k} \right)^{k+1} + \left( \frac{e}{k} \right)^{k+2} + \ldots \]

\[ = \left( \frac{e}{k} \right)^k \left[ 1 + \frac{e}{k} + \left( \frac{e}{k} \right)^2 + \ldots \right] \]

\[ = \left( \frac{e}{k} \right)^k \frac{1}{1 - \frac{e}{k}} \]

\[ \text{Telescopic} \]

\[ \text{Tail Probability} \]

\[ i \geq k \]
Value of $k$ for which Tail Probability is small

When $n=n/2$, Tail Probability is

$$\left(\frac{e}{n^{1/2}}\right)^{n/2} \cdot \frac{1}{1 - 2e/n} \approx \frac{2e^{n/2}}{n}$$

When $n=\sqrt{n}$, Tail Probability is

$$\left(\frac{e}{\sqrt{n}}\right)^{\sqrt{n}} \cdot \frac{1}{1 - e/\sqrt{n}} \approx \left(\frac{e}{\sqrt{n}}\right)^{\sqrt{n}}$$

When $k=\log n$ or more Tail Probability is

$$\frac{e^{\log n}}{\log n} = \left(\frac{1}{\log n}\right)^{\log n} \leq \left(\frac{1}{2}\right)^{\log n} = \frac{1}{n}$$