

# Alpha-Beta Search

```
function AlphaBetaDecision(state) returns an action
  v = MaxValue(state, -infinity, +infinity)
  return the a in Actions(state) leading to a successor state with utility v.
```

```
function MaxValue(state, alpha, beta) returns a utility value
  if TerminalTest(state) then return Utility(state)
  v = -infinity
  for a, s in Successors(state) do
    v = Max(v, MinValue(s, alpha, beta))
    if v >= beta then return v
    alpha = Max(alpha, v)
  return v
```

```
function MinValue(state, alpha, beta) returns a utility value
  if TerminalTest(state) then return Utility(state)
  v = +infinity
  for a, s in Successors(state) do
    v = Min(v, MaxValue(s, alpha, beta))
    if v <= alpha then return v
    beta = Min(beta, v)
  return v
```

MAX

A1 ~~5~~ 3

MIN

B1 3

B2  $\leq 2$

B3 ~~14~~ ~~5~~ 2

C1

C2

C3

C4

C5

C6

C7

C8

C9

**3**

**12**

**8**

**2**

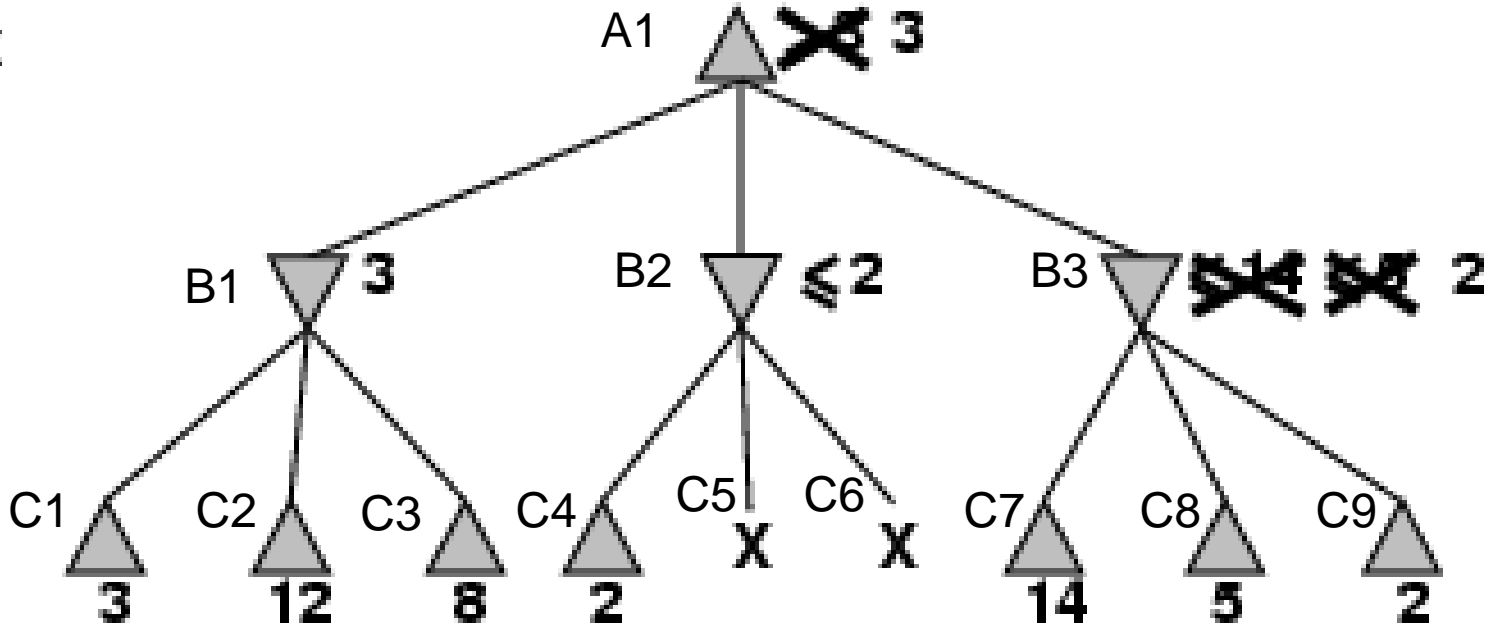
**X**

**X**

**14**

**5**

**2**



- $\text{MaxValue}(A1, -\infty, +\infty)$   
TerminalState(A1)? No  
 $v = -\infty$   
 $\text{Successors}(A1) = \{B1, B2, B3\}$   
 $s = B1$   
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\infty, +\infty)$

state = A1  
alpha = -infinity  
beta = +infinity  
v = -infinity  
s = B1

- $\text{MinValue}(B1, -\infty, +\infty)$

state = B1  
alpha = -infinity  
beta = +infinity

- $\text{MaxValue}(A1, -\infty, +\infty)$ 
  - TerminalState(A1)? No
  - $v = -\infty$
  - Successors(A1) = {B1, B2, B3}
  - $s = B1$
  - $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\infty, +\infty)$

$\text{state} = A1$   
 $\alpha = -\infty$   
 $\beta = +\infty$   
 $v = -\infty$   
 $s = B1$
  
- $\text{MinValue}(B1, -\infty, +\infty)$ 
  - TerminalState(B1)? No
  - $v = +\infty$
  - Successors(B1) = {C1, C2, C3}
  - $s = C1$
  - $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C1, -\infty, +\infty) = 3$

$\text{state} = B1$   
 $\alpha = -\infty$   
 $\beta = +\infty$   
 $v = +\infty$   
 $s = C1$

- $\text{MaxValue}(A1, -\infty, +\infty)$   
TerminalState(A1)? No  
 $v = -\infty$   
Successors(A1) = {B1, B2, B3}  
 $s = B1$   
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\infty, +\infty)$

state = A1  
alpha = -infinity  
beta = +infinity  
v = -infinity  
s = B1

- $\text{MinValue}(B1, -\infty, +\infty)$   
TerminalState(B1)? No  
 $v = +\infty$   
Successors(B1) = {C1, C2, C3}  
 $s = C1$   
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C1, -\infty, +\infty) = 3$   
 $v = \text{Min}(v, 3) = \text{Min}(+\infty, 3) = 3$   
 $v \leq \alpha$ ?  $3 \leq -\infty$ ? No

state = B1  
alpha = -infinity  
beta = +infinity  
v = 3  
s = C1

- MaxValue(A1, -infinity, +infinity)
  - TerminalState(A1)? No
  - $v = -\text{infinity}$
  - Successors(A1) = {B1, B2, B3}
  - $s = B1$
  - MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

state = A1  
 alpha = -infinity  
 beta = +infinity  
 $v = -\text{infinity}$   
 $s = B1$

- MinValue(B1, -infinity, +infinity)
  - TerminalState(B1)? No
  - $v = +\text{infinity}$
  - Successors(B1) = {C1, C2, C3}
  - $s = C1$
  - MaxValue(s, alpha, beta) = MaxValue(C1, -infinity, +infinity) = 3
  - $v = \text{Min}(v, 3) = \text{Min}(+\text{infinity}, 3) = 3$
  - $v \leq \alpha$ ?  $3 \leq -\text{infinity}$ ? No
  - $\beta = \text{Min}(\beta, v) = \text{min}(+\text{infinity}, 3) = 3$

state = B1  
 alpha = -infinity  
 beta = 3  
 $v = 3$   
 $s = C1$

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$   
TerminalState(A1)? No  
 $v = -\text{infinity}$   
Successors(A1) = {B1, B2, B3}  
 $s = B1$   
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\text{infinity}, +\text{infinity})$

state = A1  
alpha = -infinity  
beta = +infinity  
v = -infinity  
s = B1
  
- $\text{MinValue}(B1, -\text{infinity}, +\text{infinity})$   
TerminalState(B1)? No  
 $v = +\text{infinity}$   
Successors(B1) = {C1, C2, C3}  
 $s = C2$   
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C2, -\text{infinity}, 3) = 12$   
 $v = \text{Min}(v, 12) = \text{Min}(3, 12) = 3$   
 $v \leq \alpha$ ?  $3 \leq -\text{infinity}$ ? No  
 $\beta = \text{Min}(\beta, v) = \text{min}(3, 3) = 3$

state = B1  
alpha = -infinity  
beta = 3  
v = 3  
s = C2



- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$   
 $\text{TerminalState}(A1)?$  No  
 $v = -\text{infinity}$   
 $\text{Successors}(A1) = \{B1, B2, B3\}$   
 $s = B1$   
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\text{infinity}, +\text{infinity})$

$\text{state} = A1$   
 $\alpha = -\text{infinity}$   
 $\beta = +\text{infinity}$   
 $v = -\text{infinity}$   
 $s = B1$
  
- $\text{MinValue}(B1, -\text{infinity}, +\text{infinity})$   
 $\text{TerminalState}(B1)?$  No  
 $v = +\text{infinity}$   
 $\text{Successors}(B1) = \{C1, C2, C3\}$   
 $s = C3$   
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C3, -\text{infinity}, 3) = 8$   
 $v = \text{Min}(v, 8) = \text{Min}(3, 12) = 3$   
 $v \leq \alpha? 3 \leq -\text{infinity}? \text{ No}$   
 $\beta = \text{Min}(\beta, v) = \text{min}(3, 3) = 3$

$\text{state} = B1$   
 $\alpha = -\text{infinity}$   
 $\beta = 3$   
 $v = 3$   
 $s = C3$

- MaxValue(A1, -infinity, +infinity)
  - TerminalState(A1)? No
  - $v = -\text{infinity}$
  - Successors(A1) = {B1, B2, B3}
  - $s = B1$
  - MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

state = A1  
 alpha = -infinity  
 beta = +infinity  
 $v = -\text{infinity}$   
 $s = B1$

- MinValue(B1, -infinity, +infinity)
  - TerminalState(B1)? No
  - $v = +\text{infinity}$
  - Successors(B1) = {C1, C2, C3}
  - $s = C3$
  - MaxValue(s, alpha, beta) = MaxValue(C3, -infinity, 3) = 8
  - $v = \text{Min}(v, 8) = \text{Min}(3, 12) = 3$
  - $v \leq \text{alpha}$ ?  $3 \leq -\text{infinity}$ ? No
  - $\text{beta} = \text{Min}(\text{beta}, v) = \text{min}(3, 3) = 3$

state = B1  
 alpha = -infinity  
 beta = 3  
 $v = 3$   
 $s = C3$

return v: returns 3

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$

TerminalState(A1)? No

$v = -\text{infinity}$

Successors(A1) = {B1, B2, B3}

$s = B1$

$\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\text{infinity}, +\text{infinity}) = 3$

$v = \text{Max}(v, \text{MinValue}(B1, -\text{infinity}, +\text{infinity})) = \text{Max}(-\text{infinity}, 3) = 3$

state = A1

alpha = -infinity

beta = +infinity

v = 3

s = B1

- $\text{MaxValue}(A1, -\infty, +\infty)$ 
  - $\text{TerminalState}(A1)?$  No
  - $v = -\infty$
  - $\text{Successors}(A1) = \{B1, B2, B3\}$
  - $s = B1$
  - $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B1, -\infty, +\infty) = 3$
  - $v = \text{Max}(v, \text{MinValue}(B1, -\infty, +\infty)) = \text{Max}(-\infty, 3) = 3$
  - $v \geq \beta? 3 \geq +\infty?$  No
  - $\alpha = \text{Max}(\alpha, v) = \text{Max}(-\infty, 3) = 3$

state = A1  
alpha = 3  
beta = +infinity  
v = 3  
s = B1

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$ 
  - $\text{TerminalState}(A1)?$  No
  - $v = -\text{infinity}$
  - $\text{Successors}(A1) = \{B1, B2, B3\}$
  - $s = B2$
  - $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B2, 3, +\text{infinity})$

$\text{state} = A1$   
 $\alpha = 3$   
 $\beta = +\text{infinity}$   
 $v = 3$   
 $s = B2$

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$   
TerminalState(A1)? No  
 $v = -\text{infinity}$   
 $\text{Successors}(A1) = \{B1, B2, B3\}$   
 $s = B2$   
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B2, 3, +\text{infinity})$   
  
 $\text{state} = A1$   
 $\alpha = 3$   
 $\beta = +\text{infinity}$   
 $v = 3$   
 $s = B2$
- $\text{MinValue}(B2, 3, +\text{infinity})$   
  
 $\text{state} = B2$   
 $\alpha = 3$   
 $\beta = +\text{infinity}$

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$   
 $\text{TerminalState}(A1)?$  No  
 $v = -\text{infinity}$   
 $\text{Successors}(A1) = \{B1, B2, B3\}$   
 $s = B2$   
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B2, 3, +\text{infinity})$

$\text{state} = A1$   
 $\alpha = 3$   
 $\beta = +\text{infinity}$   
 $v = 3$   
 $s = B2$
  
- $\text{MinValue}(B2, 3, +\text{infinity})$   
 $\text{TerminalState}(B1)?$  No  
 $v = +\text{infinity}$   
 $\text{Successors}(B2) = \{C4, C5, C6\}$   
 $s = C4$   
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C4, 3, +\text{infinity}) = 2$

$\text{state} = B2$   
 $\alpha = 3$   
 $\beta = +\text{infinity}$   
 $v = +\text{infinity}$   
 $s = C4$

- $\text{MaxValue}(A1, -\text{infinity}, +\text{infinity})$   
TerminalState(A1)? No  
 $v = -\text{infinity}$   
Successors(A1) = {B1, B2, B3}  
 $s = B2$   
 $\text{MinValue}(s, \alpha, \beta) = \text{MinValue}(B2, 3, +\text{infinity})$

state = A1  
alpha = 3  
beta = +infinity  
v = 3  
s = B2
  
- $\text{MinValue}(B2, 3, +\text{infinity})$   
TerminalState(B1)? No  
 $v = +\text{infinity}$   
Successors(B2) = {C4, C5, C6}  
 $s = C4$   
 $\text{MaxValue}(s, \alpha, \beta) = \text{MaxValue}(C4, 3, +\text{infinity}) = 2$   
 $v = \text{Min}(v, 3) = \text{Min}(+\text{infinity}, 2) = 2$   
 $v \leq \alpha$ ?  $2 \leq 3$ ? **YES**  
return v: returns 2

state = B2  
alpha = 3  
beta = +infinity  
v = 2  
s = C4