CSE 5311 Fall 2005
Sample Questions

1. Answer questions 1a to 1c pertaining to the STOOGE_SORT Algorithm given below.

   STOOGE_SORT(A,*i*,*j*)

   if *A*[*i*] > *A*[*j*]
   then exchange *A*[*i*] ↔ *A*[*j*]
   if *i*+1 ≥ *j*
   then return
   k ← ⌊(j-i+1)/3⌋
   STOOGE_SORT(A,*i*,*j*-*k*)
   STOOGE_SORT(A,*i*+*k*,*j*)
   STOOGE_SORT(A,*i*,*j*-*k*)

   a. Argue that STOOGE_SORT (*A*,1,length[*A*]) correctly sorts the input array *A*[1 .. *n*], where *n* = length[*A*].
   b. Give a recurrence for the worst-case running time of STOOGE_SORT and a tight asymptotic bound on the worst-case running time.
   c. Compare the worst-case running time of STOOGE_SORT with that of insertion sort, merge sort, heapsort, and quicksort.

2. A child wants to construct the tallest tower possible out of building blocks. She has *n* types of blocks, and an unlimited supply of blocks of each type. Each type-*i* block is a rectangular solid with linear dimensions $<x_i,y_i,z_i>$. A Block can be oriented so that any two of its three dimensions determine the dimensions of base and the other dimension is the height. In building a tower one block may be placed on top of another block as long as the two dimensions of the base of the upper block are each strictly smaller than the corresponding base dimensions of the lower block. (Blocks oriented to have equal-sized bases can not be stacked). Design an efficient algorithm to determine the tallest tower the child can build.

3. Banks often record transactions on an account in order of the times of the transactions, but many people like to receive their bank statements with checks listed in order by check number. People usually write check in order by check number. The problem of combining time-of-transaction ordering to check-number ordering is therefore the problem of sorting almost sorted input. Analyse the performance of at least three sorting algorithm to accomplish the above task. Which is the best method.

4. Let G=(V,E) be a directed weighted graph such that all the weights are positive. Let v and w be two vertices in G and $k \leq |V|$ be an integer. Design an algorithm to find the shortest path from v to w that contains exactly k edges. The edge path need not be simple.

5. The input is two strings of characters $A = a_1\ a_2 \ldots a_n$ and $B = b_1\ b_2 \ldots b_n$. Design an O(n) algorithm to determine whether B is a cyclic shift of A. In other words, the algorithm should determine whether there exists an index k, $1 \leq k \leq n$ such that $a_i = b_{(k+i) \bmod n}$, for all *i*, $1 \leq i \leq n$.

6. The largest common subsequence (LCS) of two sequences T and P is the largest sequence L such that L is a subsequence of both T and P. The smallest common supersequence (SCS) of two sequences T and P is the smallest sequence L such that both T and P are subsequences of L.
   Design efficient algorithms to find the LCS and SCS of two given sequences.
   Let $d(T,P)$ be the smallest edit distance between T and P such that no replacements are allowed (in other words, we have to insert or delete). Prove that $d(T,P) = |SCS(T,P)| - |LCS(T,P)|$, where $|SCS(T,P)|$ is the size of the smallest SCS of T and P and $|LCS(T,P)|$ is the smallest LCS of T and P.

7. An Euler circuit of an undirected graph $G(V,E)$ is a path that starts and ends at the same node and contains each edge of G exactly once.
   Show that a connected, undirected graph has an Euler circuit if and only if each node is of even degree.

8. Let G (V,E) be an undirected graph with m edges in which every node is of even degree. Give an $O(|V|)$ algorithm to construct an Euler circuit for G.

9. Design a DP algorithm for the change-making problem: Given an amount n and unlimited quantities of coins of each of the denominations $d_1, d_2, \ldots d_m$, find the smallest number of coins that add up to n or indicate that the problem does not have a solution.

10. How would you construct an optimal binary search tree for a set of n keys if all keys are equally likely to be searched for? What will be the average number of comparisons in the tree if $n = 2^k$?

11. Show how to compute the length of an LCS using only 2 $min(m,n)$ entries in the c table plus O(1) additional space. Then, show how to do this using min(m,n) entries plus O(1) additional space.

12. G = (V,E) be an undirected graph such that each vertex has an even degree. Design a linear-time algorithm to direct the edges of G such that, for each vertex, the outdegree is equal to the indegree.

13. Design an algorithm to find a vertex in a connected undirected graph whose removal does not disconnect the graph. The algorithm should run in linear time.

14. A graph is said to be bipartite if all its vertices can be partitioned into wo disjoint subsets X and Y so that every edge connects a vertex in X with a vertex in Y.

    a. Design a DFS-based algorithm for checking whether a graph is bipartite?
    b. Design a BFS-based algorithm for checking whether a graph is bipartite?

15. Consider a set S of $n \geq 2$ distinct numbers given in unsorted order.
    a. In O(n) time, determine $x,y \in S$ such that $|x-y| \geq |w-z|$ for all $w,z \in S$
    b. In O($n \log n$) time, determine $x,y \in S$ such that $x \neq y$ and $|x-y| \leq |w-z|$ for all $w,z \in S$ such that $w \neq z$.