CSE 5311 Design and Analysis of Algorithms
Fall 2007
Quiz 2 Sample Paper
Each Question carries 25 points
Answer All questions
Time: 60 Minutes

Name:
ID:

NOTE:
*NO Reference Sheets allowed for this quiz.

*Please "Turn off" Cell Phones
   – If Cell Phone rings during the quiz: Minus 25 points penalty.

Special Instructions: This is a sample set of questions.
Your quiz will have three questions:
Q1: Similar Q1 here, but the problem will be based one of matrix-chain, optimal binary search trees or 0-1 knapsack.
Q2: One (chosen by the instructor) question from the set (1, 2, 3, 4) in Exercise Set 4.
Q3: One (chosen by the instructor) question from the set (5, 6, 7) in Exercise Set 4.
The Exercise Set 4 is on pages 3 and 4 of this document

1. Apply the dynamic programming algorithm to the following instance of the matrix-chain multiplication problem. Give the recurrence formula used for creating the solution tables. Show the table used for the solution. What is the optimal parenthesization.

Matrix	A1	A2	A3	A4	A5
Size	30×35	35×15	15×5	5×10	10×20

2. What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?
   a:1   *b*:1   *c*:2   *d*:3   *e*:5   *f*:8   *g*:13   *h*:21
   Can you generalize your answer to find the optimal code when the frequencies are the first *n* Fibonacci numbers?

3. Suppose we want to replicate a file over a collection of n servers, labeled $S_1, S_2, \ldots, S_n$. To place a copy of the file at server S_i results in a placement cost of c_i, where c_i is an integer greater than 0. Now if a user requests the file from server S_i, and no copy of the file is present at S_i, then servers $S_{i+1}, S_{i+2}, \ldots, S_{i+3} \ldots$ are searched in order until a copy of the file is finally found, say at server S_j, where $j > i$. This results in an access cost of $j-i$. (Note that the lower-indexed servers $S_{i-1}, S_{i-2} \ldots$ are not consulted in this search.) The access cost is 0 if Si holds a copy of file. We will require that a copy of the file be placed at server S_n, so that all such searches will terminate, at the latest at S_n.

    We'd like to place copies of the files at servers so as to minimize the sum of placement and access costs. Formally, we say that a configuration is a choice, for each server S_i with i = 1, 2, ...n-1, of whether to place a copy of the file at S_i or not. (Recall that a copy is always placed at S_n.) The total cost of a configuration is the sum of all placement costs for servers with a copy of the file, plus the sum of all access costs associated with all n servers.

    Give a polynomial-time algorithm to find a configuration of minimum total cost.

CSE5311 Design and Analysis of Algorithms
Exercise Set 4
10/03/07
Note: You will be required to answer selected (by the instructor) questions from this set during Quiz 2 on October 10.

1. Suppose you have n video streams that need to be sent, one after another, over a communication link. Stream i consists of a total of b_i bits that need to be sent, at a constant rate, over a period of t_i seconds. You cannot send two steams at the same time, so you need to determine a schedule for the streams: an order in which to send them. Whichever order you choose, there cannot be any delays between the end of one stream and the start of the next. Suppose your schedule starts at time 0 (and therefore ends at $\sum_{i=1}^{n} t_i$, whichever order you choose.) We assume that all the values b_i and t_i are positive integers. A schedule is valid if it satisfies the constraint imposed by the link: for each natural number $t > 0$, the total number of bits you send over the time interval from 0 to t cannot exceed $r{\times}t$, where r is the link parameter in bits/sec.

    a. Claim: There exists a valid schedule if and only if each stream i satisfies $b_i \leq r{\times}t_i$. Decide whether the claim is true or false, and give a proof of either the claim or its negation.
    b. Give an algorithm that takes a set of n streams, each specified by its number and bits b_i and its duration t_i as well as the link parameter r_i and determines whether there exists a valid schedule. The running time of your algorithm should be polynomial in n.

2. What is an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers?
    a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21
    Can you generalize your answer to find the optimal code when the frequencies are the first n Fibonacci numbers?

3. Let $R(i,j)$ be the number of times that table entry m[i,j] is referenced by Matrix-Chain order in computing other table entries. Show that the total number of references for the entire table is

$$\sum_{i=1}^{n} \sum_{j=1}^{n} R(i,j) = \frac{n^3 - n}{3}$$

4. Show how to compute the length of an LCS using only 2 min(m,n) entries in the c table plus O(1) additional space. Then, show how to do this using min(m,n) entries plus O(1) additional space.

5. A child wants to construct the tallest tower possible out of building blocks. She has n types of blocks, and an unlimited supply of blocks of each type. Each type-i block is a rectangular solid with linear dimensions $<x_i,y_i,z_i>$. A Block can be oriented so that any two of its three dimensions determine the dimensions of base and the other dimension is the height. In building a tower one block may be placed on top of another block as long

as the two dimensions of the base of the upper block are each strictly smaller than the corresponding base dimensions of the lower block. (Blocks oriented to have equal-sized bases cannot be stacked). Design a dynamic programming algorithm to determine the tallest tower the child can build.

6. Suppose you are managing the construction of billboards on the Stephen Daedalus Memorial Highway, a heavily traveled stretch of road that runs west-east for M miles. The possible sites for billboards are given by numbers $x_1, x_2, \dots, x_n$, each in the interval $[0, M]$ (specifying their position along the highway measured in miles from its western end). If you place a billboard at location x_i, you receive a revenue of $r_i > 0$. Regulations imposed by the county's Highway Department require that no two billboards be within less than or equal to 5 miles of each other. You'd like to place billboards at a subset of sites so as to maximize your total revenue, subject to this restriction.

7. Suppose we want to replicate a file over a collection of n servers, labeled $S_1, S_2, \dots, S_n$. To place a copy of the file at server S_i results in a placement cost of c_i, where c_i is an integer greater than 0. Now if a user requests the file from server S_i, and no copy of the file is present at S_i, then servers $S_{i+1}, S_{i+2}, \dots, S_{i+3} \dots$ are searched in order until a copy of the file is finally found, say at server S_j, where $j > i$. This results in an access cost of $j-i$. (Note that the lower-indexed servers $S_{i-1}, S_{i-2} \dots$ are not consulted in this search.) The access cost is 0 if Si holds a copy of file. We will require that a copy of the file be placed at server S_n, so that all such searches will terminate, at the latest at S_n.

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Give a polynomial-time algorithm to find a configuration of minimum total cost.