

Combinatorial Testing

- Introduction
- Combinatorial Coverage Criteria
- Pairwise Test Generation
- Summary

Motivation

- The behavior of a software application may be affected by many factors, e.g., input parameters, environment configurations, and state variables.
- Techniques like **equivalence partitioning** and **boundary-value analysis** can be used to identify the possible values of individual factors.
- It is impractical to test all possible combinations of values of all those factors. (Why?)

Combinatorial Explosion

- Assume that an application has 10 parameters, each of which can take 5 values. How many possible combinations?

Example - sort

```
> man sort
Reformatting page. Wait... done

User Commands                               sort(1)

NAME
  sort - sort, merge, or sequence check text files

SYNOPSIS
  /usr/bin/sort [ -cmu ] [ -o output ] [ -T directory ]
    [ -y [ kmem ] ] [ -z recsz ] [ -dfiMnr ] [ -b ] [
  -t char ]
    [ -k keydef ] [ +pos1 [ -pos2 ] ] [ file... ]
  ...
```

Combinatorial Design

- Instead of testing all possible combinations, a subset of combinations is generated to satisfy some well-defined combination strategies.
- A key observation is that not every factor contributes to every fault, and it is often the case that a fault is caused by interactions among a few factors.
- Combinatorial design can dramatically reduce the number of combinations to be covered but remains very effective in terms of fault detection.

Fault Model

- A t -way interaction fault is a fault that is triggered by a certain combination of t input values.
- A **simple** fault is a t -way fault where $t = 1$; a **pairwise** fault is a t -way fault where $t = 2$.
- In practice, a majority of software faults consist of **simple** and **pairwise** faults.

Example - Pairwise Fault

```

begin
  int x, y, z;
  input (x, y, z);
  if (x == x1 and y == y2)
    output (f(x, y, z));
  else if (x == x2 and y == y1)
    output (g(x, y));
  else
    output (f(x, y, z) + g(x, y))
end

```

Expected: $x = x1$ and $y = y1 \Rightarrow f(x, y, z) - g(x, y)$; $x = x2, y = y2 \Rightarrow f(x, y, z) + g(x, y)$

Example - 3-way Fault

```

// assume  $x, y \in \{-1, 1\}$ , and  $z \in \{0, 1\}$ 
begin
  int x, y, z, p;
  input (x, y, z);
  p = (x + y) * z // should be  $p = (x - y) * z$ 
  if (p >= 0)
    output (f(x, y, z));
  else
    output (g(x, y));
end

```

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All Combinations Coverage

- Every possible combination of values of the parameters must be covered
- For example, if we have three parameters $P1 = (A, B)$, $P2 = (1, 2, 3)$, and $P3 = (x, y)$, then **all combinations coverage** requires 12 tests: $\{(A, 1, x), (A, 1, y), (A, 2, x), (A, 2, y), (A, 3, x), (A, 3, y), (B, 1, x), (B, 1, y), (B, 2, x), (B, 2, y), (B, 3, x), (B, 3, y)\}$

Each Choice Coverage

- Each parameter value must be covered in at least one test case.
- Consider the previous example, a test set that satisfies **each choice coverage** is the following: $\{(A, 1, x), (B, 2, y), (A, 3, x)\}$

Pairwise Coverage

- Given **any** two parameters, every combination of values of these two parameters are covered in at least one test case.
- A pairwise test set of the previous example is the following:

	P1	P2	P3
A	1	x	
A	2		x
A	3		x
A	-		y
B	1		y
B	2		y
B	3		y
B	-		x

T-Wise Coverage

- Given any t parameters, every combination of values of these t parameters must be covered in at least one test case.
- For example, a **3-wise coverage** requires every triple be covered in at least one test case.
- Note that **all combinations, each choice, and pairwise coverage** can be considered to be a special case of **t -wise coverage**.

Base Choice Coverage

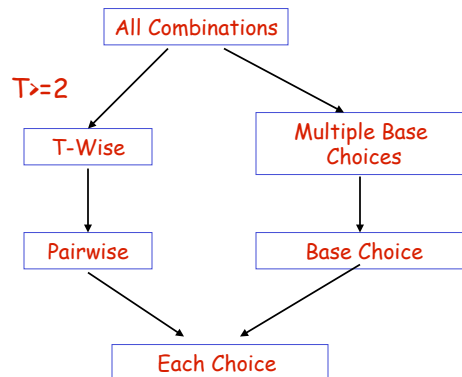
- For each parameter, one of the possible values is designated as a **base choice** of the parameter
- A **base test** is formed by using the **base choice** for each parameter
- **Subsequent tests** are chosen by holding all base choices constant, except for one, which is replaced using a non-base choice of the corresponding parameter:

	P1	P2	P3
A	1	x	x
B	1	x	x
A	2	x	x
A	3	x	x
A	1	y	

Multiple Base Choices Coverage

- At least one, and possibly more, base choices are designated for each parameter.
- The notions of a **base test** and **subsequent tests** are defined in the same as **Base Choice**.

Subsumption Relation



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Why Pairwise?

- Many faults are caused by the interactions between two parameters
 - 92% **statement** coverage, 85% **branch** coverage
- Not practical to cover all the parameter interactions
 - Consider a system with n parameter, each with m values. How many interactions to be covered?
- A trade-off must be made between test effort and fault detection
 - For a system with 20 parameters each with 15 values, **pairwise testing** only requires less than 412 tests, whereas **exhaustive testing** requires 15^{20} tests.

Example (1)

Consider a system with the following parameters and values:

- parameter A has values A1 and A2
- parameter B has values B1 and B2, and
- parameter C has values C1, C2, and C3

Example (2)

<u>A</u>	<u>B</u>	<u>C</u>
A1	B1	C1
A1	B2	C2
A2	B1	C3
A2	B2	C1
A2	B1	C2
A1	B2	C3

<u>A</u>	<u>B</u>	<u>C</u>
A1	B1	C1
A1	B2	C1
A2	B1	C2
A2	B2	C3
A2	B1	C1
A1	B2	C2
A1	B1	C3

<u>A</u>	<u>B</u>	<u>C</u>
A1	B1	C1
A1	B2	C1
A2	B1	C2
A2	B2	C2
A2	B1	C1
A1	B1	C2
A1	B1	C3
A2	B2	C3

The IPO Strategy

- First generate a pairwise test set for the first two parameters, then for the first three parameters, and so on
- A pairwise test set for the first n parameters is built by extending the test set for the first $n - 1$ parameters
 - **Horizontal growth:** Extend each existing test case by adding one value of the new parameter
 - **Vertical growth:** Adds new tests, if necessary

Algorithm IPO $H(T, p_i)$

```

Assume that the domain of  $p_i$  contains values  $v_1, v_2, \dots,$  and  $v_q$ ;
 $\pi = \{ \text{pairs between values of } p_i \text{ and values of } p_1, p_2, \dots, \text{ and } p_{i-1} \}$ 
if (  $|T| \leq q$  )
  for  $1 \leq j \leq |T|$ , extend the  $j^{\text{th}}$  test in  $T$  by adding value  $v_j$ 
  and remove from  $\pi$  pairs covered by the extended test
else
  for  $1 \leq j \leq q$ , extend the  $j^{\text{th}}$  test in  $T$  by adding value  $v_j$  and
  remove from  $\pi$  pairs covered by the extended test;
  for  $q < j \leq |T|$ , extend the  $j^{\text{th}}$  test in  $T$  by adding one value of
   $p_i$  such that the resulting test covers the most number of
  pairs in  $\pi$ , and remove from  $\pi$  pairs covered by the
  extended test

```

Algorithm IPO_V(T, π)

```

let  $T'$  be an empty set;
for each pair in  $\pi$ 
  assume that the pair contains value  $w$  of  $p_k$ ,  $1 \leq k < i$ , and value  $u$  of  $p_i$ ;
  if ( $T'$  contains a test with "-" as the value of  $p_k$  and  $u$  as the value of  $p_i$ )
    modify this test by replacing the "-" with  $w$ 
  else
    add a new test to  $T'$  that has  $w$  as the value of  $p_k$ ,  $u$  as the value of  $p_i$ , and "-" as the value of every other parameter;
 $T = T \cup T'$ 

```

Example Revisited

Show how to apply the IPO strategy to construct the pairwise test set for the example system.

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Summary

- Combinatorial testing makes an excellent **trade-off** between test effort and test effectiveness.
- **Pairwise testing** can often reduce the number of dramatically, but it can still detect faults effectively.
- The IPO strategy constructs a pairwise test set incrementally, one parameter at a time.
- In practice, some combinations may be invalid from the domain semantics, and must be excluded, e.g., by means of constraint processing.