This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2007 proceedings.

Complexity and Error Propagation of Localization Using Interferometric Ranging

Rui Huang, Gergely V. Záruba, and Manfred Huber
Computer Science and Engineering Department
The University of Texas at Arlington
416 Yates, 300NH, Arlington, TX 76019
Email: rxh1725@omega.uta.edu, zaruba@uta.edu, huber@uta.edu

***Abstract*— An interferometric ranging technique has been recently proposed as a possible way to localize ad hoc and sensor networks. Compared to the more common techniques such as received signal strength, time of arrival, and angle of arrival ranging, interferometric ranging has the advantage that the measurement could be highly precise. However, localization using interferometric ranging is difficult as it requires a large number of measurement readings. In this paper, we provide a formal proof of this difficulty in terms of algorithmic complexity. Furthermore, we propose an iterative algorithm that calculates node locations from a set of seeding anchors, gradually building a more global localization solution. Compared to previous localization algorithms, which treat localization as a global optimization problem, the iterative algorithm is a distributed algorithm that is simple to implement in larger networks. More importantly, the iterative algorithm allows us to study the error propagation behavior of localization using interferometric ranging. Using simulations, we validate the performance of the iterative algorithm in terms of localization error and coverage.**

I. Introduction

Location discovery is emerging as one of the more important tasks in ad hoc and sensor networks as it has been observed and shown that (semi-) accurate location information can greatly improve the performance of tasks such as routing, energy conservation, or maintaining network security. A direct way to obtain location information is to install global positioning system (GPS) receivers on each node in the network. However, this is currently impractical as GPS receivers are still relatively bulky, expensive, power-hungry, and require clear line of sight to several earth-bound satellites (i.e., making indoor usage impossible). In sensor networks, devices are imagined as small as possible and operating on a very restricted power source; thus it may not be feasible to install GPS receivers into all sensor nodes. Therefore, the localization problem arises in that there is a need to determine the location of all nodes based on the location of a limited subset of localized nodes (also called *anchors*).

An excellent overview of the localization problem is given in [1]. In general, the problem is often classified into subproblems based on the sensory data available between a pair of nodes. Most common types of sensory data include received signal strength indication (RSSI) [2]–[4], time of arrival (TOA) ranging using ultrasound [5], angle of arrival (AOA) [6], connectivity-only (or range-free) [7]–[9], or some combination of the above [10], [11]. In most cases, the nodes collaborate to derive the location based on the anchor locations and the sensory data observed. Unfortunately, it has been shown that the localization problem is NP-Complete when localized from either ranging [12], angle [13] or unit-disk connectivity [14]. Thus, the localization problem is treated as an optimization problem, for which a distributed solution is more desirable.

Other than those widely studied sensory types, a new sensory type called interferometric ranging has recently been proposed [15]. Interferometric ranging is a "widely used technique in both radio and optical astronomy to determine the precise angular position of celestial bodies as well as objects on the ground [16]." Its original design is to work with large scale systems where objects are thousands of miles apart. However, there have been recent advances in hardware design that allow interferometric ranging to be performed on cheaper hardware, making it a promising new technique for localizing ad hoc and sensor networks. Interferometric ranging exploits the property that the relative phase offset between two receivers determines the distances between the two senders. By synchronizing the transmission at the two senders, the distance difference (also called the *q-range*) can be measured very accurately using interferometric ranging.

Definition 1.1: A *q-range* obtained from interferometric ranging from two senders A and B, and two receivers C and D is the distance difference $d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC} + e$, where e is the measurement error (Figure 1).

Definition 1.2: Given a q-range d_{ABCD}, A, B, C and D are also referred as the *components* of the q-range, in which A and B are the *senders* and C and D are the *receivers*.

Note that based on the above definitions, within each q-range A is interchangeable with B since both are senders, and C is interchangeable with D since both are receivers. Thus, $d_{ABCD} = -d_{BACD} = -d_{ABDC} = d_{BADC}$.

A major advantage of interferometric ranging is that the measurement could be extremely accurate compared to noise-prone RSSI readings. In a recent experiment [15], in which 16 nodes are deployed in a 4x4 grid over a 18x18 meters flat grassy area with no obstruction, the maximum q-range error was shown to be around 0.1 meters while the medium error was less than 0.04 meters. However, interferometric ranging is more difficult to implement partially due to the following reasons:

1) Precise time synchronization is needed at all four com-

1-4244-0353-7/07/$25.00 ©2007 IEEE

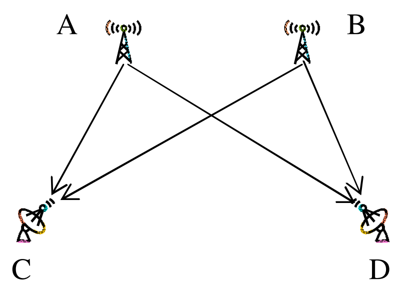


Fig. 1. The interferometric ranging measurement of the q-range $d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC}$. Here, node A and B are the senders, and node C and D are the receivers.

ponents of a q-range.

2) Frequencies of the transmissions need to be precisely calibrated.
3) A significantly larger number of measurements are needed for localization than using direct ranging techniques.
4) Since each measurement involves four nodes, more collaboration is required between nodes.

Those difficulties rooted in the physical characteristics of interferometric ranging devices affect the algorithmic design of the localization algorithm. In this paper, we concentrate on the algorithmic aspects of the problem, and in particular, the last two difficulties. In other words, we only consider how to localize the network from a set of given q-ranges, and we do not consider how those q-ranges are obtained. A detailed overview of the physical characteristics of interferometric ranging is given in [15].

We will start by reviewing some fundamental complexity results on localization using interferometric ranging. We show in Section III that there is a polynomial time algorithm that checks for a necessary condition under which a network can be localized. We also show that the localization problem itself is NP-Complete when using interferometric ranging as the measurement. In Section IV, we further derive a sufficient condition that guarantees a unique localization of a node based on local interferometric readings. Using the condition, we propose an iterative localization algorithm that localizes the network from a small number of seeding anchors. The performance of the localization algorithm and its error propagation behavior are validated using simulations in Section V.

II. Previous Works

The large number of measurements required for localization using interferometric ranging are illustrated by the following theorem (and proof) given in [16].

Theorem 2.1: In a network of n nodes, there is a maximum of $n(n-3)/2$ independent interferometric measurements that can be obtained.

Theorem 2.1 shows the number of measurements available using interferometric measurements is $O(n^2)$, which is significant higher than with RSSI and AOA ranging ($O(n)$). Considering the localization problem in relative coordinates, for a network of n nodes there are $2n-3$ unknowns in 2 dimensions and $3n-6$ unknowns in 3 dimensions [1]. Thus, the smallest network that can be localized using interferometric measurements is a fully-connected network with a population of $n = 6$, where there are 9 independent measurements available to cover 9 unknowns.

Furthermore, for interferometric ranging not all measurements are useful. Some measurements are dependent on others, and only independent measurements are useful in localization. For instance, for the four nodes A, B, C and D in Figure 1, if all nodes are completely connected (i.e., any two can be the senders or the receivers), then there are only two independent q-ranges, e.g., $d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC}$ (when A and B are the senders) and $d_{ADBC} = d_{AC} - d_{DC} + d_{DB} - d_{AB}$ (when A and D are the senders). All other q-ranges are dependent upon those two, that is, they can be expressed as linear combinations of those two.

Given a set of interferometric measurements (i.e., q-ranges), a localization algorithm attempts to find the sensor locations that satisfy the measurements. There have been a limited number of localization algorithms proposed for interferometric ranging. A generic algorithm approach was taken in [15]. An algorithm was proposed in [17] that uses both interferometric and RSSI ranging. Both algorithms try to optimize for a global solution given an entire set of interferometric measurements. Intuitively, finding a global solution to the localization problem is often difficult because of the large search space and the large number of constraints given by the interferometric measurements. Thus, it is desirable to find the solutions in some subspaces first and then incrementally build up to the global solution. This paper makes use of an iterative approach to localize the network, which was first introduced in [12].

III. Complexity Results

In this section, we provide some complexity results on localization using interferometric ranging. We will show when using interferometric readings as the measurement the complexity of the localization problem is NP-Complete. In the remainder of this section dealing with the complexity, we assume that the q-range measurement error e is insignificant, and all q-ranges give the precise distance difference. We will reconsider the measurement error in the subsequent sections.

First, we extend Theorem 2.1 to give a polynomial time algorithm that determines the number of independent interferometric readings.

Theorem 3.1: Given a network of n nodes and a set of interferometric readings (q-ranges), there is a polynomial time algorithm that determines how many of them are independent (the set's dimension).

Proof: We start by denoting each node in the network with an integer ID starting from 0 to $n-1$, and let A, B, C and D be variables containing a unique ID. Considering a

[1] This is because the relative coordinates are invariant under translation, rotation and reflection. Thus, in 2 dimensions, we have $2n-3$ degrees of freedom, where translation, rotation and reflection each reduce one degree of freedom.

vector space consisting of all possible q-ranges, the algorithm needs to identify the total number of independent q-ranges from a given set of vectors $\{d_{ABCD}\}$. To do this, one can use the Gaussian elimination method *if* the given vectors can be written as some linear combination of a set of basis vectors. The classification algorithm given in [16] provides a way to accomplish this, as follows.

Given a vector $d_{ABCD} = d_{AD} - d_{BD} + d_{BC} - d_{AC}$ in a vector space, it has been shown in [16] that any vector d_{ABCD} should satisfy the condition $A < B, A < C < D, B \neq C, B \neq D$ in order to be independent since $d_{ABCD} + d_{BACD} = 0$ and $d_{ABCD} - d_{CDAB} = 0$. Thus, we can convert any q-ranges not satisfying the above condition to a q-range that does. Furthermore, any such vector d_{ABCD} that satisfies the above condition belongs to one of the following six classes:

- Class 0: $\{012D|2 < D\}$
- Class 1: $\{0B1D|1 < B < D\}$
- Class 2: $\{01CD|2 < C < D\}$
- Class 3: $\{0B1D|1 < D < B\}$
- Class 4: $\{0BCD|1 < B, 1 < C < D, B \neq C, B \neq D\}$
- Class 5: $\{ABCD|0 < A < B, A < C < D, B \neq C, B \neq D\}$

Among the six classes, Class 0 and Class 1 form a basis set that only contains independent vectors. Vectors in Class 2 through 5 can be written as linear combinations of those in the first two classes as follows:

- Class 2: $d_{01CD} = -d_{012C} + d_{012D}$
- Class 3: $d_{0B1D} = -d_{01DB} + d_{0D1B}$
- Class 4: $d_{0BCD} = -d_{0B1C} + d_{0B1D}$
- Class 5: $d_{ABCD} = -d_{0ACD} + d_{0BCD}$

Using the above algorithm, any given q-range in the set $\{d_{ABCD}\}$ can be written as a linear combination of the q-ranges in Class 0 and Class 1 (i.e., a basis set), with -1, 1, or 0 as the coefficient at each term.

Given a set of N q-range vectors, we construct a M-by-N matrix $\mathbb{A}$, where M is the total number of Class 0 and Class 1 vectors. For every q-range i, we run the above reduction algorithm to reduce it to a linear combination of Class 0 and Class 1, and then insert the coefficients (-1, 1, or 0) into the ith column of the matrix $\mathbb{A}$. We can then use Gaussian elimination on matrix $\mathbb{A}$ (or some other techniques in linear algebra to find the rank), which will indicate the number of independent columns of $\mathbb{A}$. Since for a network of n nodes there are $2n - 3$ unknowns in 2 dimensions and $3n - 6$ unknowns in 3 dimensions, we can compare the total count with $2n - 3$ or $3n - 6$ to determine whether the network is localizable in 2 dimensions or 3 dimensions.

For each q-range in a total of N q-ranges, the algorithm to find the coefficients in a column of $\mathbb{A}$ runs in constant time. The complexity of constructing the matrix $\mathbb{A}$ is therefore $O(N)$. From the constructed matrix $\mathbb{A}$, the complexity of running Gaussian elimination is $O(N^3)$. Thus, the overall complexity is $O(N + N^3) = O(N^3)$, polynomial time. ■

As argued in the previous section, the problem of localizing a network of n nodes in 2 dimensions involves $2n - 3$ unknowns. Therefore, if there are at least $2n - 3$ independent q-ranges, then we will have a system of $2n - 3$ independent equations to sufficiently solve for the $2n-3$ unknowns. Thus, the polynomial time algorithm enables us to check for a necessary condition of network localizablity under interferometric readings. However, since each q-range is not a linear but a quadratic equation, having $2n - 3$ independent q-ranges is only a necessary but not sufficient condition of network localizablity. Since testing the network localizablity using edge weights (distances) is a known NP-Complete problem [18], it is likely that network localizablity using interferometric ranging is also NP-Complete.

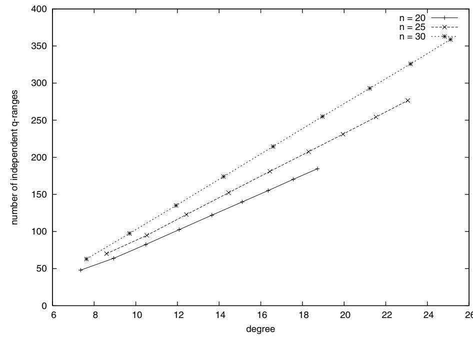


Fig. 2. In a randomly placed unit disk graph of size n, the number of independent q-ranges increase linearly to the network connectivity (average node degree).

The results above assume a general graph. In reality, ad hoc and sensor networks resemble unit disk graphs. How many independent q-ranges are we expecting in unit disk graphs then? Simulation on randomly deployed unit disk graphs show that the number of independent q-ranges increases linearly with the network connectivity (average nodal degree) as depicted in Figure 2.

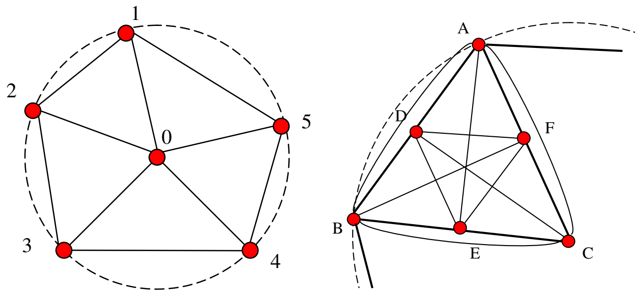
The following theorem shows that even with a known localizable network, the localization itself is NP-Complete when using interferometric ranging as the measurement.

Theorem 3.2: Given a network that is localizable using a set of interferometric readings (q-ranges), the actual localization of the network is an NP-Complete problem.

Proof: It is obvious that when a solution instance (certificate) is given (i.e., when all node locations are known), there is a polynomial time algorithm to validate such an instance against the q-ranges. Thus, the localization problem is in NP. We now show the problem is NP-Complete.

We reduce from the realization problem of wheel graphs. A wheel graph, W_n, is a graph of n nodes in which (without losing generality due to node numbering) nodes 1 through $n-1$ form a cycle (not necessarily on a circle), and node 0 (hub) is connected to all nodes. The edges on the cycle are called the *rim edges*, and edges connecting from the hub are called *spokes*. Figure 3(a) shows such a graph with $n = 6$.

It has been shown in [12] that it is NP-Complete to localize the wheel graph W_n when the edge weights (including spokes and rim edges) are known. To show that it is also NP-Complete to localize a network using interferometric readings (q-ranges),



(a) Wheel Graph W_6 (b) Reduction from the Wheel Graph

Fig. 3. Wheel graph and its reduction.

we construct a polynomial time reduction from W_n. We then claim that the reduced graph, called $\hat{W}_n$, is localizable using the q-ranges obtainable from the edge weights of W_n. And finally we show that such reduction leads to the conclusion that localizing using q-ranges is NP-Complete.

The reduction from W_n to $\hat{W}_n$ works as follows. Observe that W_n consists of a sequence of triangles formed by two spokes and a rim edge. On every edge of each triangle, we create an additional node equal distance away from both end points. We then add additional edges to completely connect all six nodes (three original nodes and three newly created nodes) within the triangle. For instance, in Figure 3(b), we create three new nodes D, E, and F within $\triangle ABC$ such that $d_{AD} = d_{DB} = \frac{d_{AB}}{2}$, $d_{BE} = d_{EC} = \frac{d_{BC}}{2}$ and $d_{AF} = d_{FC} = \frac{d_{AC}}{2}$. The subgraph consisting of those six nodes are completely connected. Clearly, the reduction step is polynomial in n.

We claim that the graph $\hat{W}_n$ is localizable in 2 dimensions using interferometric ranging. To support our claim we need to determine how many interferometric readings (q-ranges) are present in $\hat{W}_n$. First, observe that by our construction, the weight of *every* newly created edge in the triangle subgraph can be determined geometrically from three edge weights d_{AB}, d_{BC} and d_{AC} that are given by the original W_n graph. Thus, within each triangle subgraph of $n = 6$, we have *all* available q-ranges for a complete graph of $n = 6$. Conversely, from those available q-ranges, we can derive the original edge weights d_{AB}, d_{BC} and d_{AC}. For instance, we can calculate d_{AC} from the following four q-ranges:

$$\begin{aligned} d_{AECB} &= d_{AB} - d_{EB} + d_{EC} - d_{AC} \\ &= d_{AB} - \frac{d_{BC}}{2} + \frac{d_{BC}}{2} - d_{AC} \\ &= d_{AB} - d_{AC} \end{aligned} \tag{1}$$

$$\begin{aligned} d_{CDAB} &= d_{BC} - d_{DB} + d_{AD} - d_{AC} \\ &= d_{BC} - \frac{d_{AB}}{2} + \frac{d_{AB}}{2} - d_{AC} \\ &= d_{BC} - d_{AC} \end{aligned} \tag{2}$$

$$\begin{aligned} d_{BADF} &= d_{BF} - d_{AF} + d_{AD} - d_{BD} \\ &= d_{BF} - \frac{d_{AC}}{2} + \frac{d_{AB}}{2} - \frac{d_{AB}}{2} \\ &= d_{BF} - \frac{d_{AC}}{2} \end{aligned} \tag{3}$$

$$\begin{aligned} d_{BECF} &= d_{BF} - d_{EF} + d_{EC} - d_{BC} \\ &= d_{BF} - \frac{d_{AB}}{2} + \frac{d_{BC}}{2} - d_{BC} \\ &= d_{BF} - \frac{d_{AB}}{2} - \frac{d_{BC}}{2} \end{aligned} \tag{4}$$

Combining (1) with (2), and (3) with (4), we have

$$d_{AECB} + d_{CDAB} = d_{AB} + d_{BC} - 2 \cdot d_{AC} \tag{5}$$

$$\frac{d_{AC}}{2} = -d_{BECF} + d_{BADF} - \frac{d_{AB}}{2} - \frac{d_{BC}}{2} \tag{6}$$

Combining (5) with (6), and solving for d_{AC}, we have

$$d_{AC} = \frac{-2 \cdot d_{BECF} + 2 \cdot d_{BADF} - d_{AECB} - d_{CDAB}}{3}$$

It has been shown in [12] that W_n is localizable when all of its edge weights are given. Since, using the q-ranges of $\hat{W}_n$ available to us, we can drive every edge weight of W_n, W_n is localizable using the q-ranges. If W_n is localizable using q-ranges, so is $\hat{W}_n$ because by our reduction the location of every newly added node in $\hat{W}_n$ can be uniquely determined from the location of the existing nodes in W_n. Thus, $\hat{W}_n$ is localizable using q-ranges.

If there is a polynomial time algorithm, $\mathcal{A}$, that performs the actual localization of $\hat{W}_n$ using q-ranges, the original wheel graph W_n can then be localized under edge weights by running our polynomial time reduction to produce $\hat{W}_n$ and then running $\mathcal{A}$ to localize $\hat{W}_n$, all in polynomial time. However, since localizing the wheel graph W_n using edge weights is NP-Complete as shown in [12], $\mathcal{A}$ does not exist. Thus, localization using interferometric ranging (q-ranges) is NP-Complete. ■

IV. Iterative Localization Using Interferometric Ranging

Based on the above complexity result, it is clear that localization using interferometric ranging is fundamentally intractable. Any algorithm that solves this problem will need to be some form of a heuristic (e.g., described as an optimization process). However, even as an optimization problem, the problem is difficult because of the large search space. In this section, we try to provide an optimization solution by localizing the nodes incrementally based on a sufficient (but not necessary) condition of node-localizability in term of interferometric ranging.

Lemma 4.1: A node i can be localized using interferometric ranging under the following two conditions:

1) i is a component of at least three mutually independent q-ranges, **and**
2) all other three components in each q-range are localized.

Proof: Let node i be a component in the three q-ranges. For each of the three q-ranges, let the other three components be node A, B and C. Thus, the q-range is in one of the following forms, depending on whether i is a sender or a receiver:

- $d_{ABiC} = d_{AC} - d_{BC} + d_{Bi} - d_{Ai}$
- $d_{iABC} = d_{iC} - d_{AC} + d_{AB} - d_{iB}$

Since A, B and C are already localized, the distances d_{AB}, d_{AC} and d_{BC} can be calculated from the node locations. Thus, the q-range can be reduced to the following:

- $c_1 = d_{ABiC} + d_{BC} - d_{AC} = d_{Bi} - d_{Ai}$
- $c_2 = d_{iABC} + d_{AC} - d_{AB} = -d_{iB} + d_{iC}$

Here, c_1 and c_2 are two constant values. Thus, each q-range reduces to a partial (either a left-side or a right-side) hyperbola on which the location of node i resides. Since the three q-ranges are mutually independent, each of the partial hyperbolas they generate is unique. Ignoring the rare cases such as when the hyperbolas overlap, the intersection of three unique partial hyperbolas is a unique point. Thus, node i can be localized. ■

Lemma 4.2: Any three q-ranges with node i as a common receiver (sender) are independent if

1) each q-range has a distinct pair of senders (receivers), **and**
2) there are in total at least four distinct senders (receivers).

Proof: First consider the trivial case where the dependency is in two q-ranges. Two types of dependencies exist in this case: $d_{ABCD} + d_{BACD} = 0$ and $d_{ABCD} - d_{CDAB} = 0$. Based on the condition given, $d_{ABCD} + d_{BACD} = 0$ would not happen because they do not have a distinct pair of senders. $d_{ABCD} - d_{CDAB} = 0$ would not happen due to the lack of a common receiver (sender).

Now consider the case when the three q-ranges are dependent. Since there are four distinct senders (receivers) in the three q-ranges, not *all* of the four senders (receivers) can appear multiple times in the q-ranges. Let j be the sender (receiver) that appears only once in the q-ranges. Thus, there is only one q-range that includes the distance between node i and j, d_{ij}. However, since j only appears once, the other two q-ranges do not include d_{ij}. Since d_{ij} is a unique term, the q-range that includes j cannot be written as a linear combination of the other two.

Thus, all three q-ranges must be independent. ■

Lemma 4.1 and 4.2 give the condition under which a node can be localized using interferometric ranging from its neighbors. If such condition is satisfied, the node can be localized with only the local neighborhood information without the complexity of a global optimization problem. Once the node is localized, its location information can be used to further localize other nodes. This gives an iterative localization algorithm shown as Algorithm 1, which is similar to the iterative trilateration protocol (ITP) proposed in [12], however with different conditions.

Algorithm 1 Iterative Localization Using Interferometric Ranging

```
Require: every node knows its 1-hop neighbor
  for all Localized nodes do
    broadcast its location and 1-hop neighbor set
  end for
  for all Unlocalized nodes i do
    S_senders ← φ
    Receive broadcasts and construct local connectivity map
    Find nodes s1, s2 and r such that e_{s1,i}, e_{s2,i}, e_{s1,r}, and
    e_{s2,r} exist in the local map
    if (s1, s2) ∉ S_senders and (s2, s1) ∉ S_senders then
      add (s1, s2) to S_senders
    end if
    if S_senders contains at least 3 pairs and at least 4 distinct
    senders then
      negotiate to obtain q-ranges using each pair as senders
      or receivers
      determine its location
      broadcast its location and 1-hop neighbor set
    end if
  end for
```

Algorithm 1 requires all nodes to have their 1-hop connectivity information, which can be collected by observing "Hello" messages from the neighbors. Before starting the localization process, a small number of localized nodes (anchors) need to be deployed. Due to the condition listed in Lemma 4.1 and 4.2, those anchors need to be close to each other such that the nearby unlocalized nodes can be localized. The anchors then broadcast their locations and 1-hop connectivity information. When an unlocalized node hears the broadcast, it builds a local connectivity map, which is needed to validate the condition required by Lemma 4.1 and 4.2. In particular, when there are at least three distinct pairs of potential interferometric senders (s_1, s_2), each of which shares a common receiver r, then the node can be localized. When the node detects that the localization condition is satisfied, it contacts the potential sender pairs and the corresponding receiver to schedule an interferometric reading. When at least three readings are obtained, the node computes its location, and then broadcast it to its neighbors so that the newly discovered location can be used to localize the neighbors in the next round. The localization algorithm continues until all nodes are localized or the next round does not produce any newly localized node.

Simplicity is the real advantage of the above iterative algorithm. Instead of trying to solve for a global solution for all unlocalized nodes in the network at once, the algorithm computes the locations based on the local information only and then progressively builds up a global solution. The algorithm is also distributed in nature and can be implemented directly on each sensor. However, since the conditions dictated by Lemmas 4.1 and 4.2 are *not* sufficient conditions for a node to be localized, the algorithm does *not* guarantee to localize a node even though the node *could* be theoretically localized. For a randomly deployed sensor network, the ratio of the localizable nodes to the population using the iterative algorithm is a function of the network density. Fortunately, our simulations show that this ratio is reasonably high when compared to the localization ratio of the iterative trilateration protocol (ITP) in [12] that uses direct RSSI ranging.

To compute nodes' locations after the sufficient number of q-ranges are obtained, we run a simple simulated annealing algorithm. The simulated annealing approach is taken because i) in reality the system is often over-determined by multiple q-ranges which contain errors, and ii) the location function to be optimized is non-linear with multiple local minimums. Since the localization is performed using local q-ranges only, we are able to drastically limit the search space to be the area within the range of all components involved in the transmission. Simulations show that the simulated annealing algorithm converges quickly to the correct solution.

V. Simulation Result

To evaluate the behavior of the iterative localization algorithm using interferometric ranging, we have conducted a number of simulations in various of settings. Our simulation environment consists of a network of n sensors randomly deployed in a square area of 1 unit square. The sensors are assumed to be homogeneous, i.e., they all have the same transmission range. To start the localization process, we deploy four anchors to the center of the unit square (the anchors have the same transmission range as the sensors). The results obtained are the average of 30 independent runs.

A. Coverage and Rounds

We first look at the localization coverage of the iterative algorithm and the number of rounds executed by the algorithm. We compare the result against that of the iterative trilateration protocol (ITP) proposed in [12]. The node localization condition of the ITP is that an unlocalized node needs to be the neighbor of at least three localized nodes. The localization condition stated in Lemma 4.1 and 4.2 is stricter since it requires three independent q-ranges. At the minimum, an unlocalized node needs to neighbor with four localized nodes to satisfy such a condition [2]. Thus, it is expected that for the same network the localization coverage produced by the iterative algorithm for interferometric ranging is lower than that of the trilateration.

As validated in Figure 4(a), for a fixed transmission range, the number of nodes localized using the interferometric condition is indeed smaller than that of the trilateration condition. However, the difference is not particular great and can be overcome by increasing the network density. As shown in Figure 4(a), by increasing the node density (by increasing the population to $n = 160$), the interferometric localization condition can reach a coverage similar to what trilateration reaches at $n = 100$. In term of density, the network of $n = 100$ has a degree of 12 when the transmission range is set to 0.2, which results in 90% coverage under the trilateration condition. The equivalent coverage can be obtained under the interferometric condition by increasing the density to 19 degrees when using $n = 160$ with the same transmission range. The number of rounds required to complete the iterative algorithm is shown in Figure 4(b).

B. Localization Error

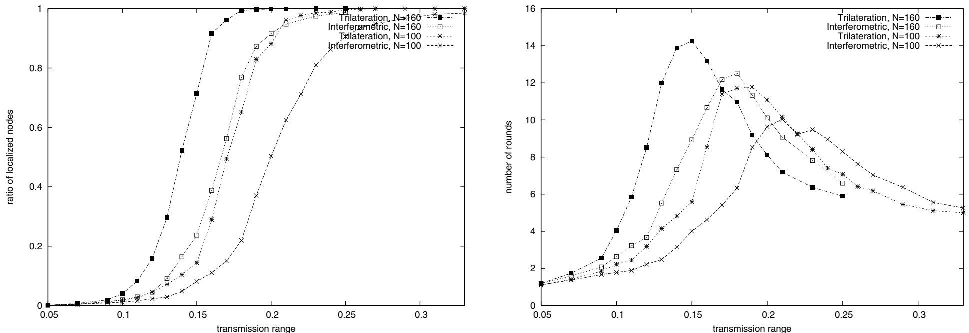
The iterative localization algorithm allows us to study the error propagation behavior of interferometric ranging. In particular, we are interested in how the error from the interferometric measurement affects the localization error and how the error is aggregated and propagated through the network. Figure 5(a) shows the average localization error at each round for a network of 100 nodes using the iterative algorithm (transmission range set to 0.25 units). A Gaussian noise of $N(0, \sigma)$ is added to the interferometric measurement. The standard deviation are in the same range as derived by the actual hardware devices in [15]. Figure 5(a) indicates a linear increase of the localization error of the nodes localized at each additional round. Thus, it is more desirable that most nodes are localized with limited rounds. The number of nodes localized at each round is shown in Figure 5(b). In our simulation scenario, most nodes are localized within 5 rounds. Nevertheless, the simulation indicates that the error propagation behavior poses a significant constraint on how effectively the interferometric measurement can be applied to the localization problem. In order to achieve more precise localization, the effect of error propagation has to be controlled.

VI. Conclusion

Interferometric ranging has been recently proposed as a viable measurement type to solve the localization problem in sensor networks. However, in addition to constraints imposed to the hardware devices, interferometric ranging also imposes new challenges to the algorithmic design of localization. In this paper, we formally proved that localization using interferometric ranging is an NP-Complete problem. Compared to heuristics on direct RSSI or TOA ranging (both of which problems are also NP-Complete) it can be argued that heuristics on localization using interferometric ranging are even more difficult (however this added difficulty is polynomial). Whereas each direct ranging measurement is a function of two locations, the interferometric measurement is a function of four. Thus, localization using interferometric ranging requires a considerably larger set of measurements.

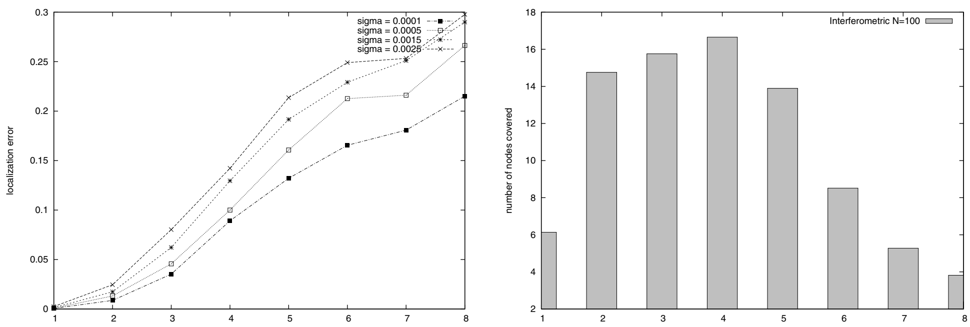
Previous interferometric localization algorithms try to optimize the solution globally either by using a generic algorithm approach [15] or by reducing the search space with additional RSSI readings [17]. The difficulty of the problem still limits their solutions to smaller networks (16 nodes in [15] and 25 nodes in [17]). The iterative algorithm proposed in this paper allows networks of larger size to be localized using interferometric ranging. However, our simulation indicated that error

[2]Three localized nodes, along with the unlocalized node, would generate a maximum of two independent q-ranges instead of three.



(a) Localization coverage. (b) The number of rounds required.

Fig. 4. Localization coverage and the number of rounds required.



(a) Localization error of the nodes localized at each round. (b) The number of nodes localized at each round.

Fig. 5. Localization error.

propagation can be a potentially significant problem. In order to localize large networks using interferometric ranging from a small set of anchors, future localization algorithms need to find a way to effectively limit the error propagation.

REFERENCES

[1] N. Patwari, A. Hero, J. Ash, R. Moses, S. Kyperountas, and N. Correal, "Locating the Nodes: Cooperative Geolocation of Wireless Sensors," *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 54-69, July 2005.

[2] P. Bahl and V.N. Padmanabhan. "RADAR: An In-Building RF-Based User Location and Tracking System," in Proceedings of *IEEE INFOCOM'00*, vol. 2, pp. 775-784, Tel Aviv, Israel, 2000.

[3] D. Niculescu and B. Nath. "Ad Hoc Positioning System (APS)," in Proceedings of *IEEE GLOBECOM'01*, vol. 5, pp. 2926 - 2931, San Antonio, 2001.

[4] R. Huang and G. V. Záruba. "Location Tracking in Mobile Ad Hoc Networks Using Particle Filters," in Proceedings of *ADHOC-NOW*, pp. 85-98, 2005.

[5] A. Savvides, C. C. Han, and M. Srivastava. "Dynamic Fine-Grained Localization in Ad Hoc Networks of Sensors," in Proceedings of *MOBICOM'01*, pp. 166 - 179, Rome, Italy, 2001.

[6] K. Chintalapudi, R. Govindan, G. Sukhatme and A. Dhariwal. "Ad-hoc Localization Using Ranging and Sectoring," in Proceedings of *IEEE INFOCOM '04*, vol. 4, pp. 2662- 2672, April 2004.

[7] N. Bulusu, J. Heidemann and D. Estrin. "GPS-less Low Cost Outdoor Localization for Very Small Devices," *IEEE Personal Communications Magazine*, vol. 7, no. 5, pp. 28–24, October 2000.

[8] T. He, C. Huang, B. M. Blum,J. A. Stankovic, and T. F. Abdelzaher. "Range-Free Localization Schemes in Large Scale Sensor Networks," in Proceedings of *ACM MOBICOM'03*, pp. 81 - 95, 2003.

[9] L. Hu and D. Evans. "Localization for Mobile Sensor Networks," in Proceedings of *ACM MOBICOM'04*, pp. 45 - 57, 2004.

[10] X. Li, H. Shi and Y. Shang. "A Partial-Range-Aware Localization Algorithm for Ad-hoc Wireless Sensor Networks," in Proceedings of *IEEE International Conference on Local Computer Networks (LCN'04)*, vol. 00, pp. 77 - 83, 2004.

[11] R. Huang and G. V. Záruba. "Incorporating Data from Multiple Sensors for Mobile Ad Hoc Networks Localization," submitted to *IEEE TMC*, 2006.

[12] T. Eren, D. Goldenberg, W. Whiteley, Y. R. Yang, A. S. Morse, B. D. O. Anderson, and P. N. Belhumeur. "Rigidity, Computation, and Randomization of Network Localization," in Proceedings of *IEEE INFOCOM'04*, vol. 4, pp. 2673 - 2684, Hong Kong, China, Apr. 2004.

[13] J. Bruck, J. Gao, and A. Jiang. "Localization and Routing in Sensor Networks by Local Angle Information," in Proceedings of *ACM MOBIHOC'05*, pp. 181 - 192, Urbana-Champaign, IL, USA, 2005.

[14] F. Kuhn, T. Moscibroda, and R. Wattenhofer. "Unit Disk Graph Approximation," in Proceedings of *Workshop on Discrete Algorithms and Methods for Mobile Computing and Communications (DIAL-M)*, pp. 17 - 23, Philadelphia, PA, USA, 2004.

[15] M. Maróti, B. Kusý, G. Balogh, P. V olgyesi, A. Nádas, K. Molnár, S. Dóra, and A. Lédeczi. "Radio Interferometric Geolocation," in Proceedings of *3rd International Conference on Embedded Networked Sensor Systems (SenSys)*, pp. 1-12, San Diego, California, USA, Nov. 2005.

[16] B. Kusý, M. Maróti, G. Balogh, P. V olgyesi, J. Sallai, A. Nádas, A. Lédeczi and L. Meertens. "Node Density Independent Localization," in Proceedings of *5th International Conference on Information Processing in Sensor Networks(IPSN)*, pp. 441 - 448, Nashville, Tennessee, USA, April 2006.

[17] N. Patwari and A. O. Hero,"Indirect Radio Interferometric Localization via Pairwise Distances," in Proceedings of *3rd IEEE Workshop on Embedded Networked Sensors (EmNets 2006)*, pp. 26-30, Boston, MA, May 30-31, 2006.

[18] J. Saxe. "Embeddability of weighted graphs in k-space is strongly NP-hard," in Proceedings of 17th Allerton Conference in Communications, Control and Computing, pp. 480-489, 1979.