Trees and Graphs

CSE 2320 – Algorithms and Data Structures
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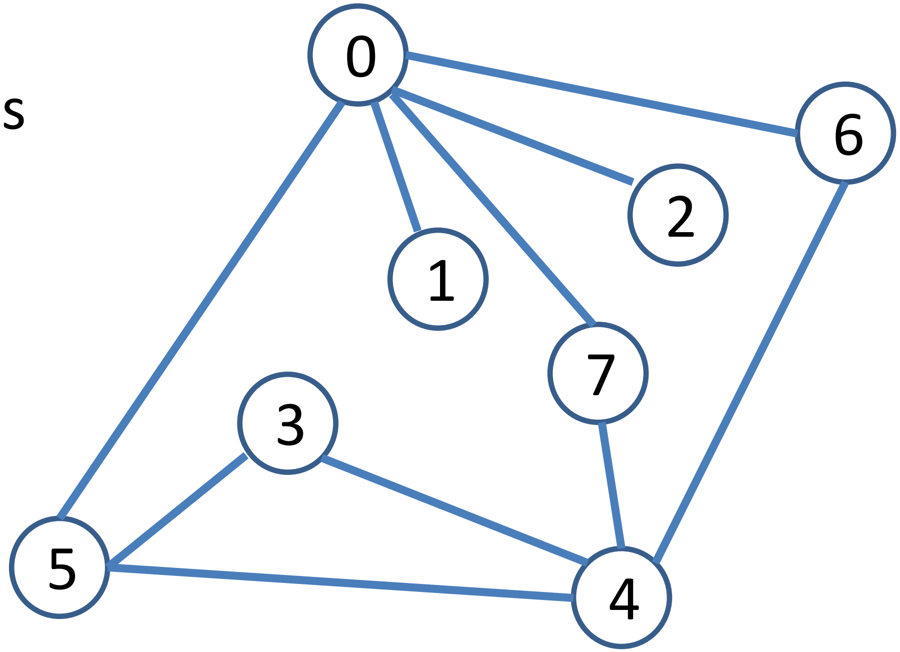
Graphs

- A graph is formally defined as:
  - A set V of **<u>vertices</u>** (also called **<u>nodes</u>**).
  - A set E of **<u>edges</u>**. Each edge is a pair of two vertices in V.
- Graphs can be directed or undirected.
- In a directed graph, edge (A, B) means that we can go (using that edge) from A to B, but **not** from B to A.
  - We can have both edge (A, B) and edge (B, A) if we want to show that A and B are linked in both directions.
- In an undirected graph, edge (A, B) means that we can go (using that edge) from both A to B and B to A.

Example: of an Undirected Graph

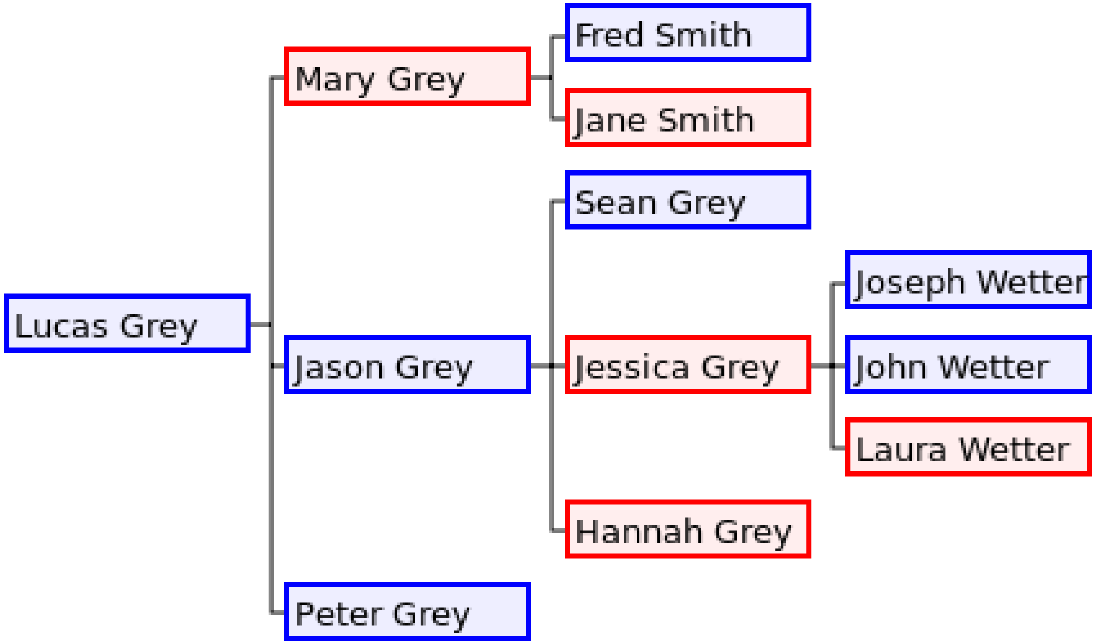
- A graph is formally defined as:
  - A set V of vertices.
  - A set E of edges. Each edge is a pair of two vertices in V.
- What is the set of vertices on the graph shown here?
  - {0, 1, 2, 3, 4, 5, 6, 7}
- What is the set of edges?
  - {(0,1), (0,2), (0,5), (0,6), (0, 7), (3, 4), (3, 5), (4, 5), (4, 6), (4,7)}.



Trees

- Trees are a natural data structure for representing several types of data.
  - Family trees.
  - Organizational chart of a corporation, showing who supervises who.
  - Folder (directory) structure on a hard drive.
  - Parsing an English sentence into its parts.

A Family Tree (from Wikipedia)



Lucas Grey

Mary Grey

Jason Grey

Peter Grey

Fred Smith

Jane Smith

Sean Grey

Jessica Grey

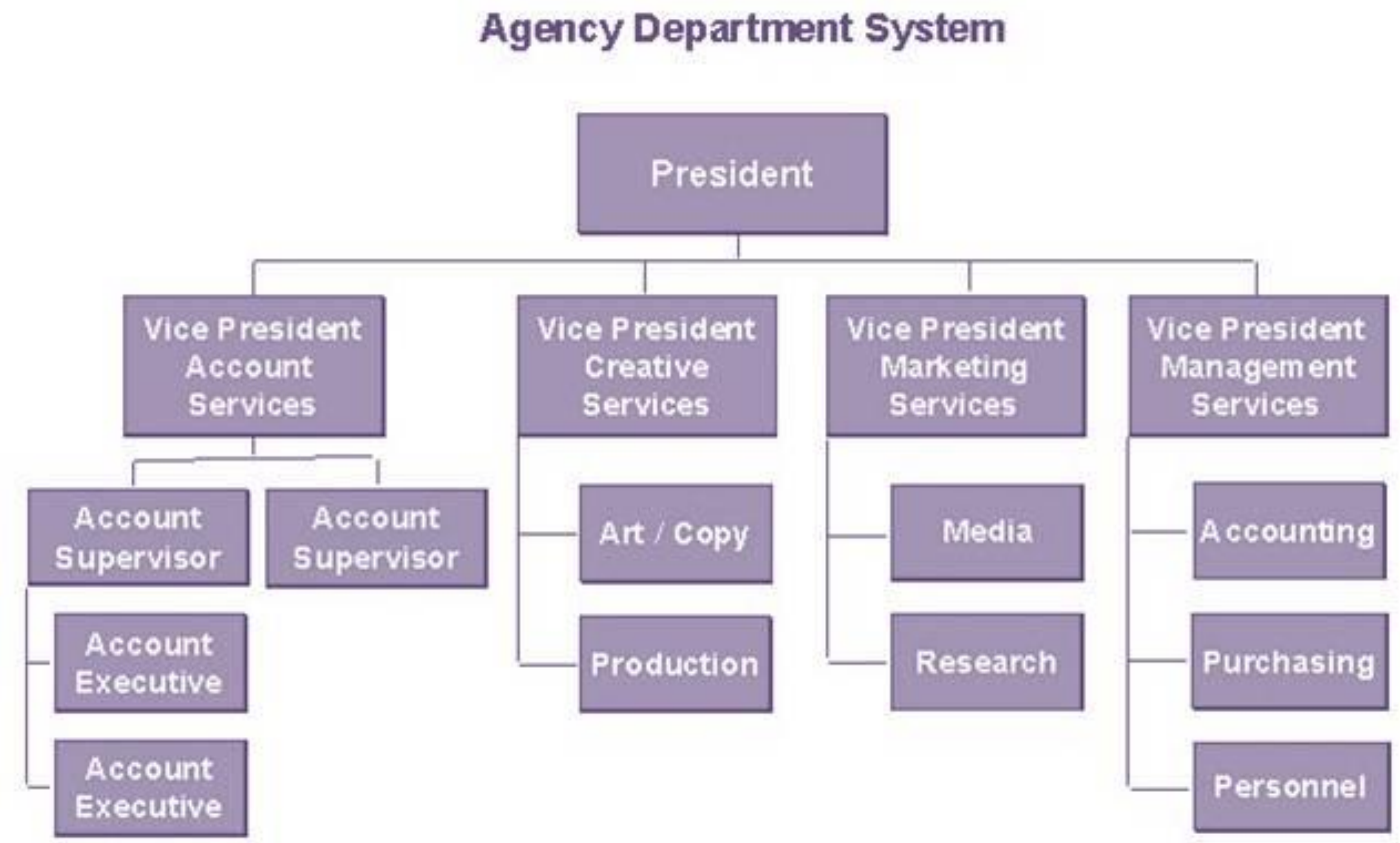
Hannah Grey

Joseph Wetter

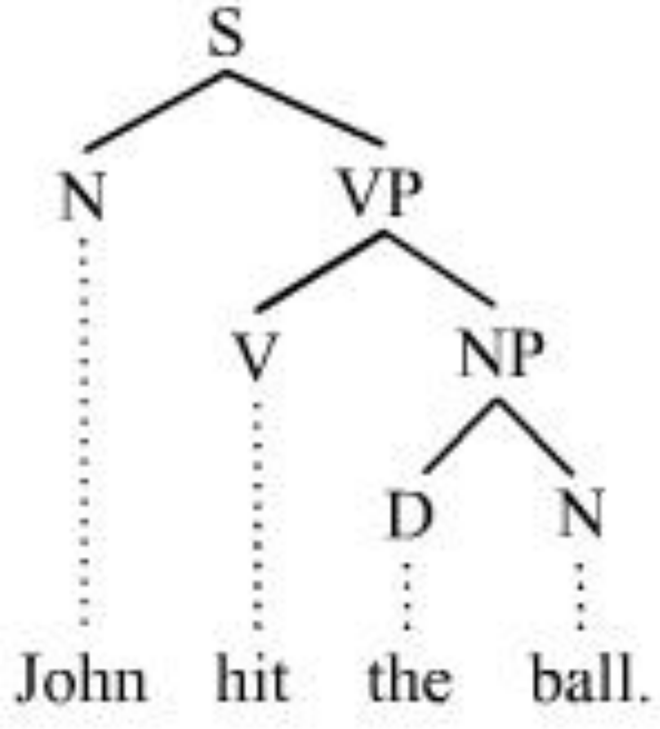
John Wetter

Laura Wetter

An Organizational Chart (from Wikipedia)



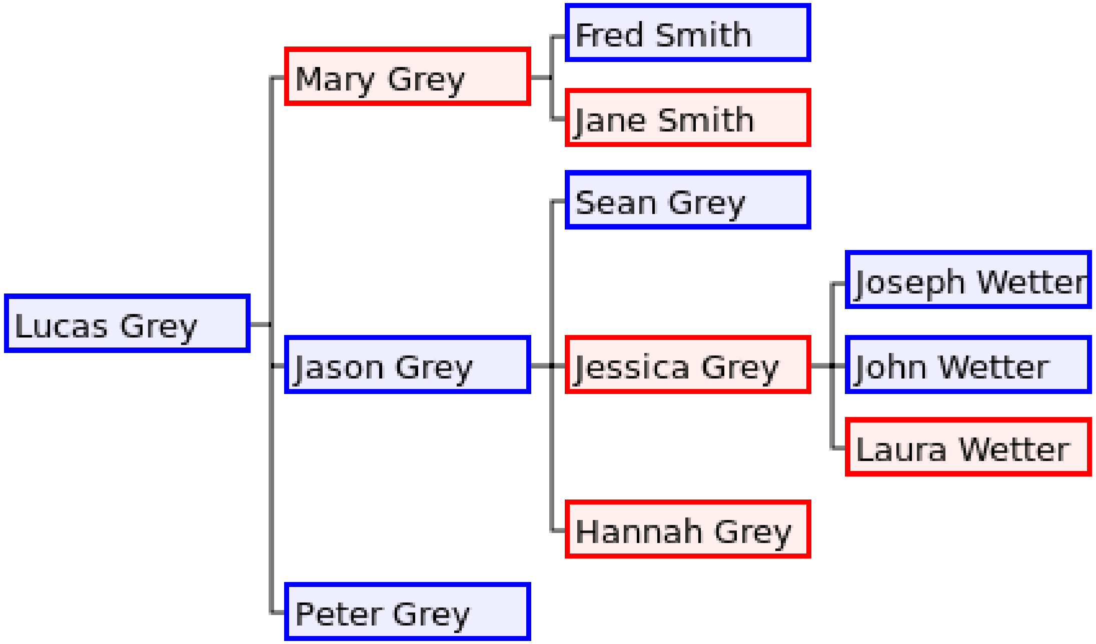
A Parse Tree (from Wikipedia)



Constituency-based parse tree

Paths

- A path in a tree is a list of distinct vertices, in which successive vertices are connected by edges.
  - No vertex is allowed to appear twice in a path.
- Example: ("Joseph Wetter", "Jessica Grey", "Jason Grey", "Hanna Grey")



Trees and Graphs

- Are trees graphs?
  - Always?
  - Sometimes?
  - Never?
- Are graphs trees?
  - Always?
  - Sometimes?
  - Never?

Trees and Graphs

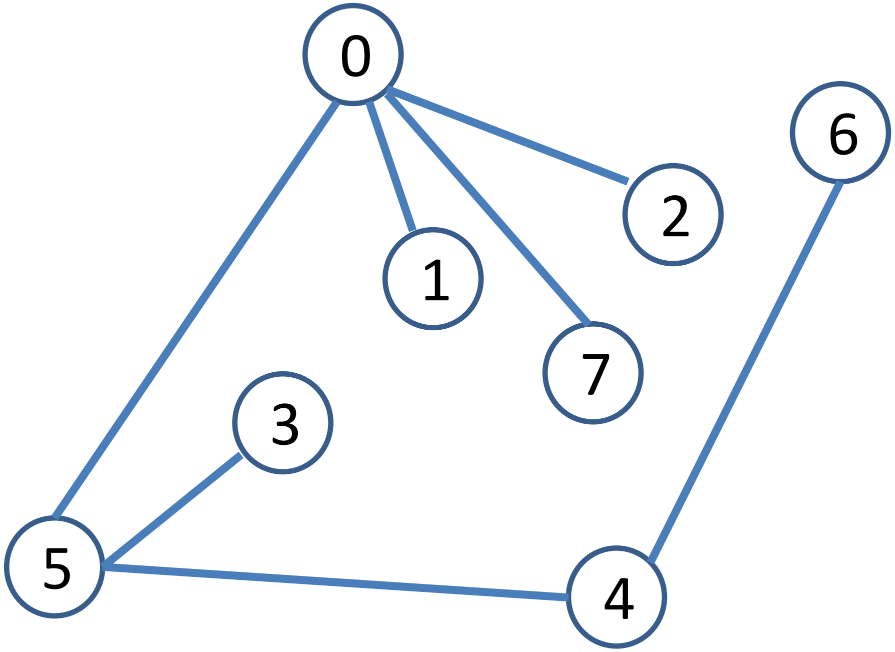
- All trees are graphs.
- Some graphs are trees, some graphs are not trees.
- What is the distinguishing characteristic of trees?
- What makes a graph a tree?

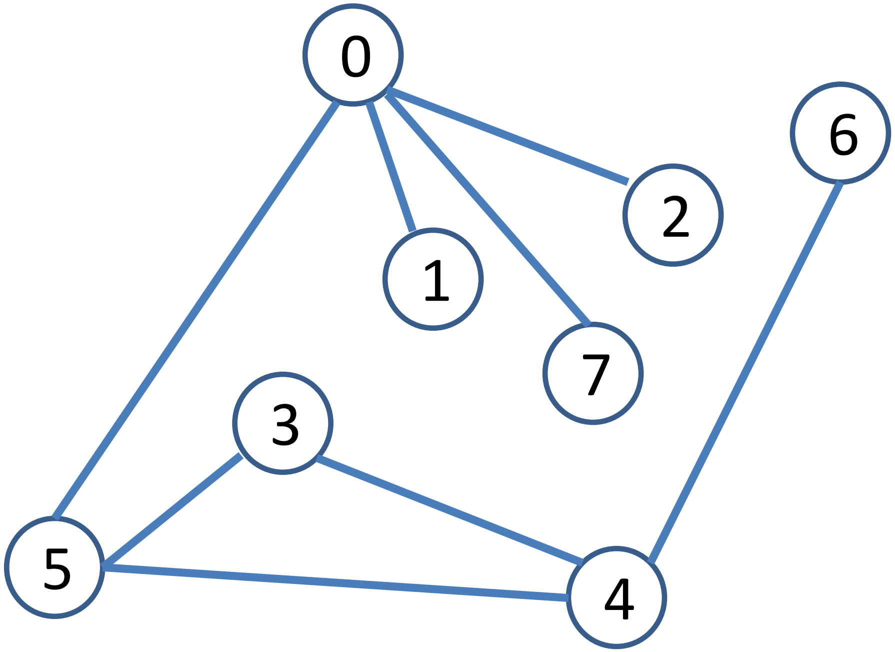
Trees and Graphs

- All trees are graphs.
- Some graphs are trees, some graphs are not trees.
- What is the distinguishing characteristic of trees?
  - What makes a graph a tree?
- A tree is a graph such that any two nodes (vertices) are connected by precisely one path.
  - If you can find two nodes that are **not** connected by any path, then the graph is not a tree.
  - If you can find two nodes that are connected to each other by more than one path, then the graph is **not** a tree.

Example

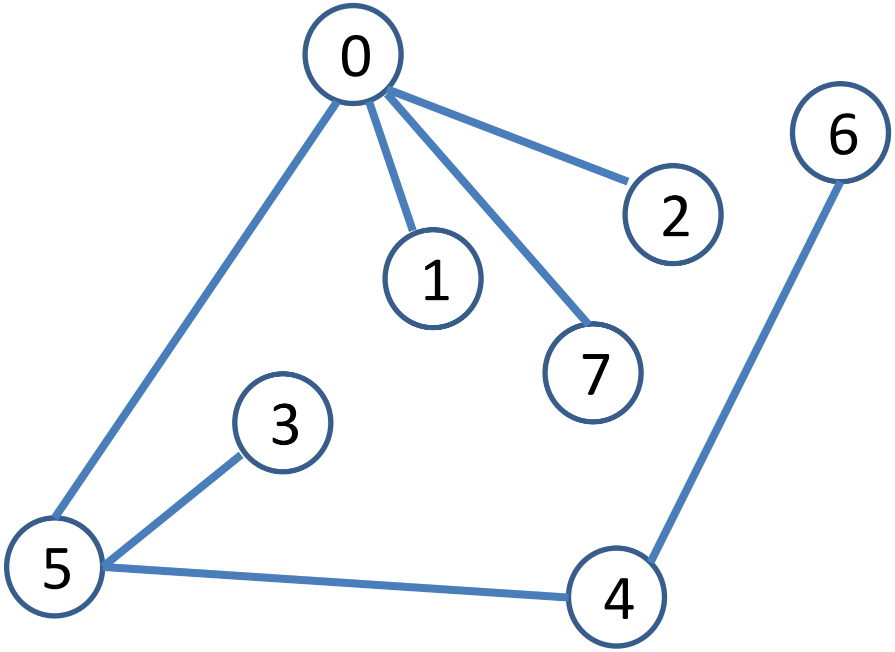
- Are these graphs trees?



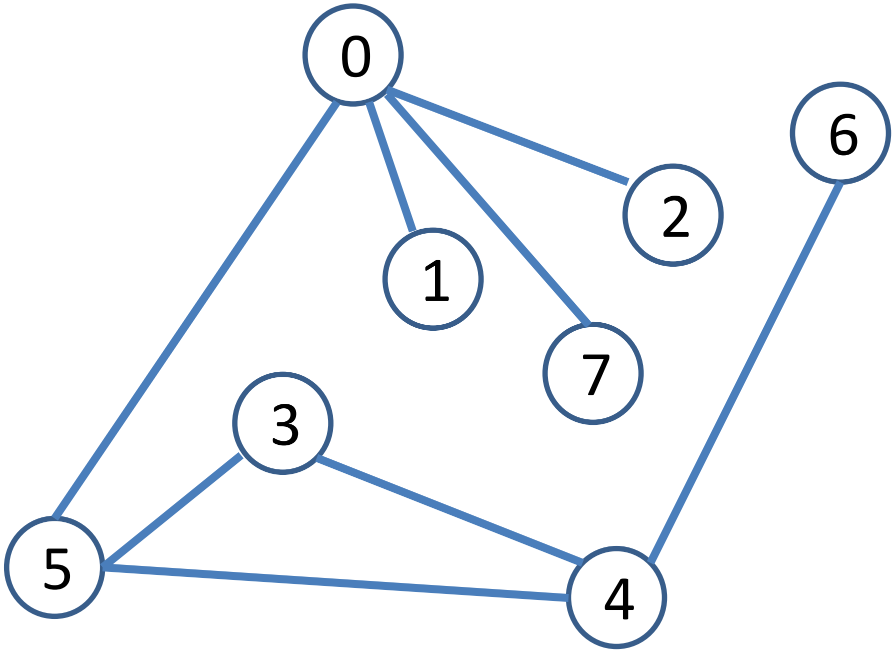


Example

- Are these graphs trees?



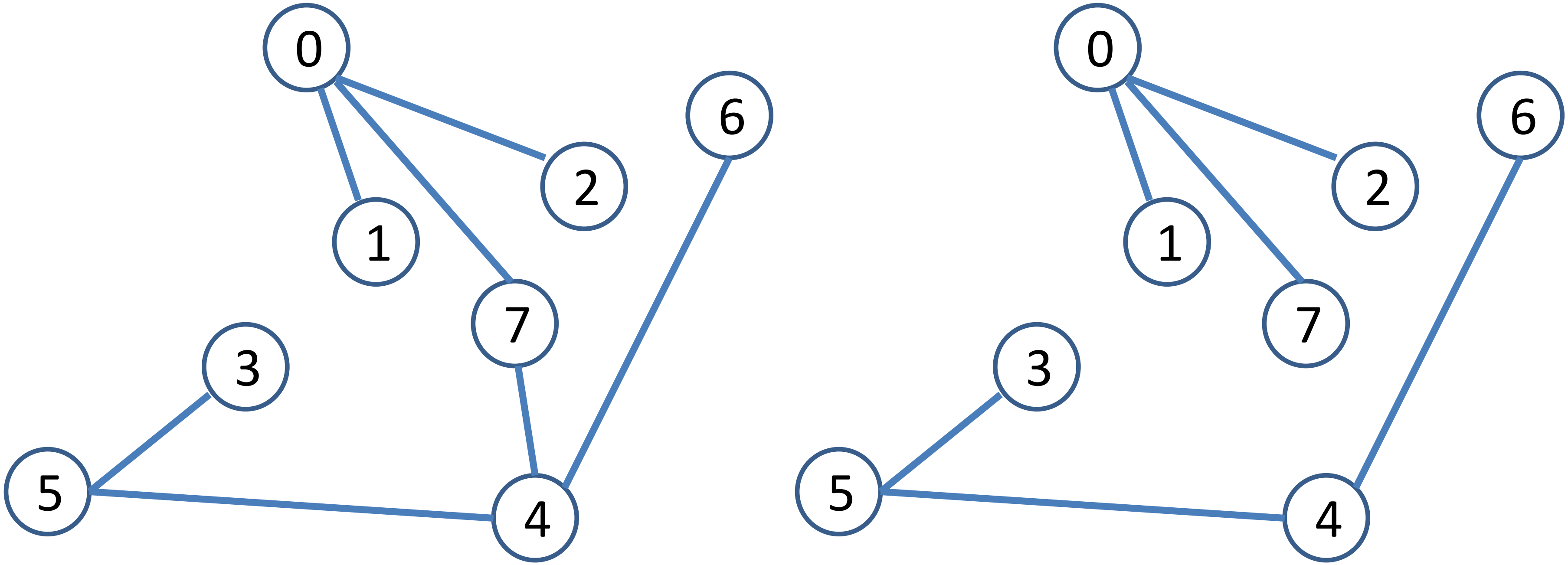
Yes, this is a tree. Any two vertices are connected by exactly one path.



No, this is not a tree. For example, there are two paths connecting node 5 to node 4.

Example

- Are these graphs trees?



Example

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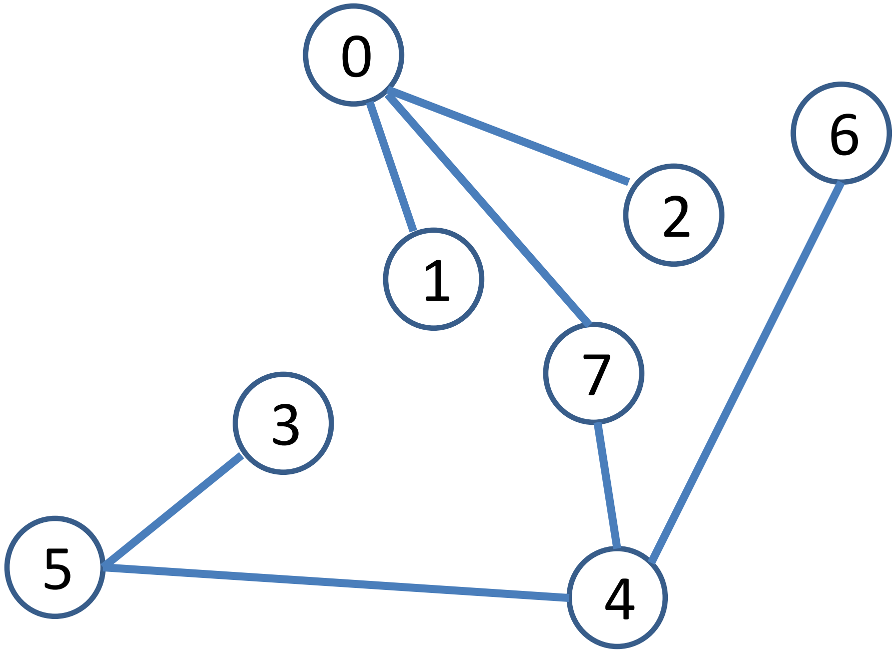
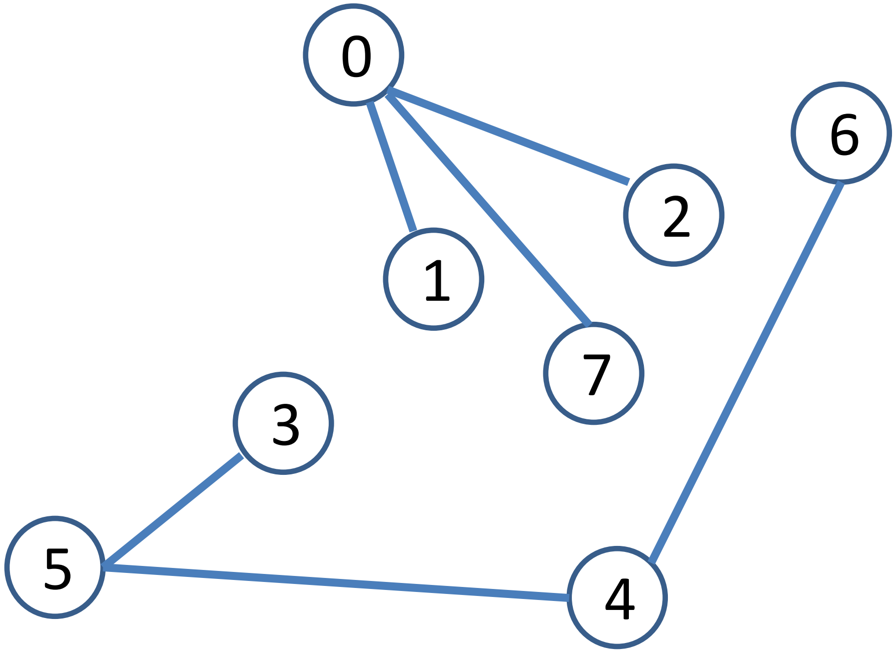


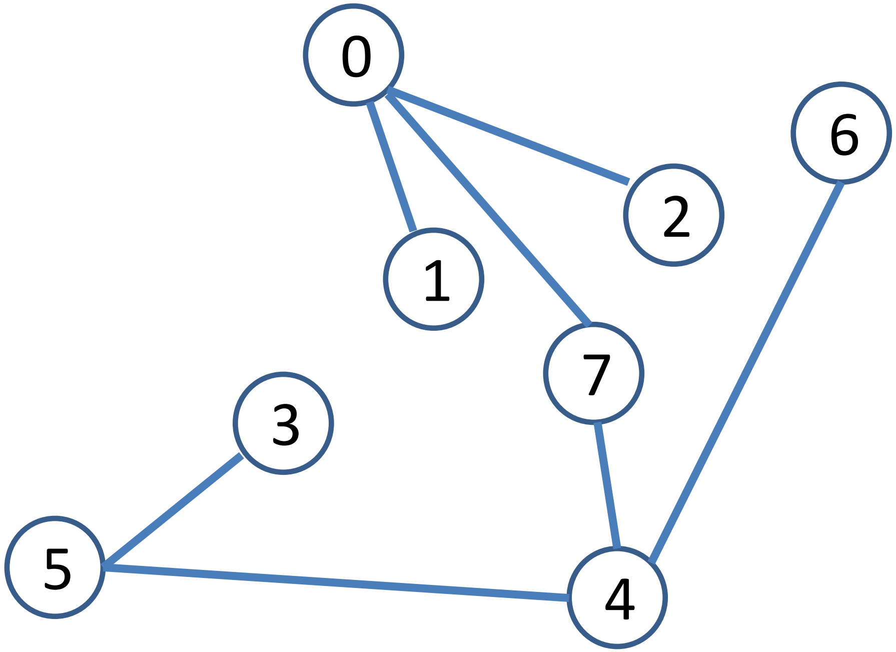
Yes, this is a tree. Any two vertices are connected by exactly one path.



No, this is not a tree. For example, there is no path connecting node 7 to node 4.

Root of the Tree

- A rooted tree is a tree where one node is designated as the root.
- Given a tree, ANY node can be the root.



Terminology

- A rooted tree is a tree where one node is explicitly designated as the root.
  - From now on, as is typical in computer science, all trees will be rooted trees
  - We will typically draw trees with the root placed at the top.
- Each node has exactly one node directly above it, which is called a **parent**.
- If Y is the parent of X, then Y is the node right after X on the path from X to the root.

Terminology

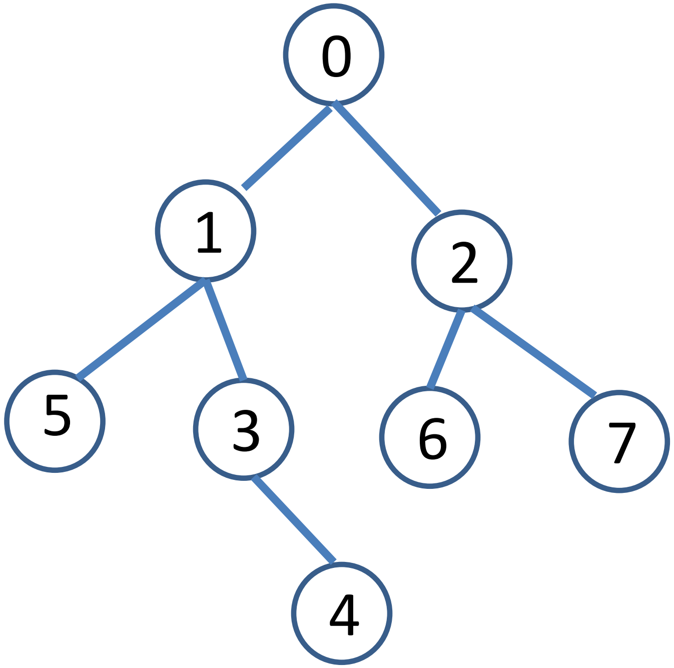
- If Y is the parent of X, then X is called a **child** of Y.
  - The root has no parents.
  - Every other node, except for the root, has exactly one parent.
- A node can have 0, 1, or more children.
- Nodes that have children are called **internal nodes** or **non-terminal nodes**.
- Nodes that have no children are called **leaves** or **terminal nodes**, or **external nodes**.

Terminology

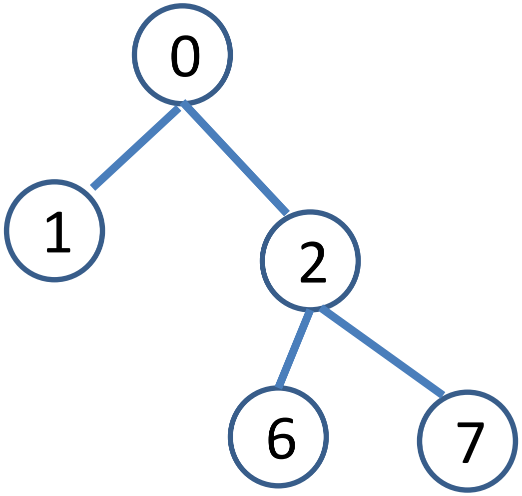
- The **level** of the root is defined to be 0.
- The **level** of each node is defined to be 1+ the level of its parent.
- The **height** of a tree is the maximum of the levels of all nodes in the tree.

M-ary Trees

- An **M-ary tree** is a tree where every node is either a leaf or it has **exactly** M children.
- Example: **binary** trees, **ternary** trees, ...



Is this a binary tree?



Is this a binary tree?

M-ary Trees

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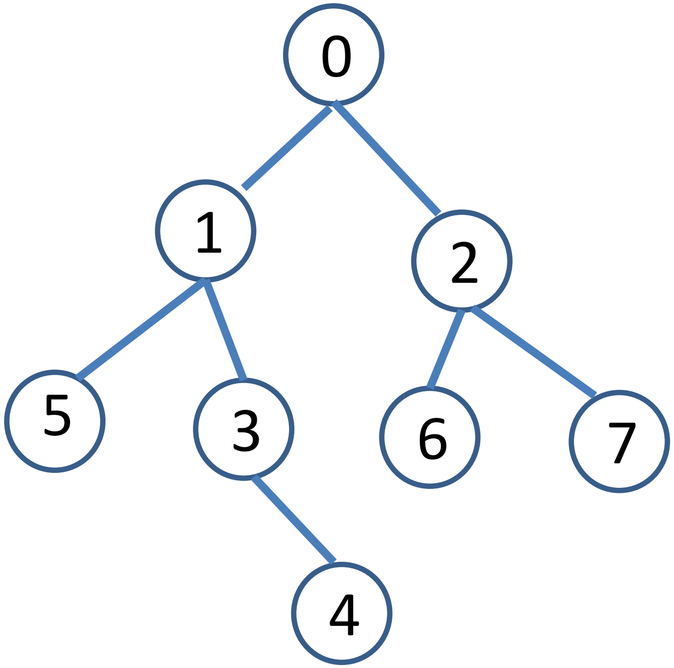
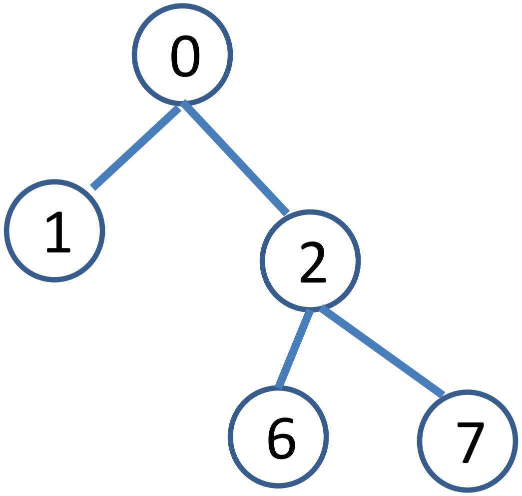


This is **not** a binary tree, node 3 has 1 child.



This is a binary tree.

Ordered Trees

- A rooted tree is called **ordered** if the order in which we list the children of each node is significant.
- For example, if we have a binary ordered tree, we will refer to the left child and the right child of each node.
- If the tree is not ordered, then it does not make sense to talk of a left child and a right child.

Properties of Binary Trees

- A binary tree with N internal notes has N+1 external nodes.
- A binary tree with N internal notes has 2N edges (links).
- The height of a binary tree with N internal nodes is at least lg N and at most N.
  - Height = lg N if all leaves are at the same level.
  - Height = N if each internal node has one leaf child.

Defining Nodes for Binary Trees

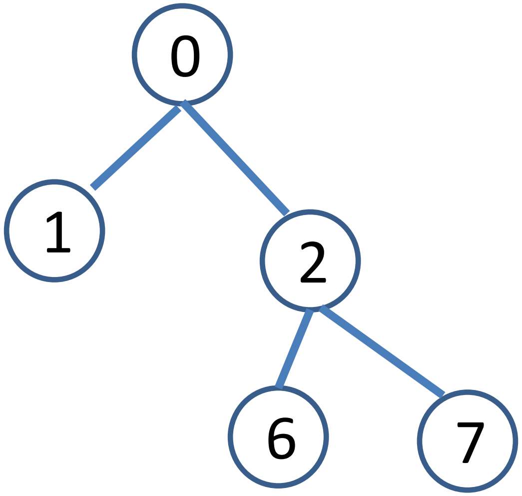
```
typedef struct node *link;
struct node
{
  Item item;
  link left;
  link right;
};
```

Traversing a Binary Tree

- **Traversing** is the process of going through each node of a tree, and doing something with that node. Examples:
  - We can print the contents of the node.
  - We can change the contents of the node.
  - We can otherwise use the contents of the node in computing something.
- We have three choices about the order in which we visit nodes when we traverse a binary tree.
  - **Preorder**: we visit the node, then its left subtree, then its right subtree.
  - **Inorder**: we visit the left subtree, then the node, then the right subtree.
  - **Postorder**: we visit the left subtree, then the right subtree, then the node.

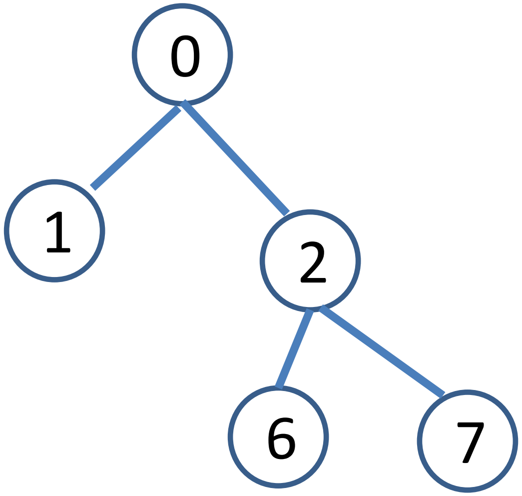
Examples

- In what order will the values of the nodes be printed if we print the tree by traversing it:
  - Preorder?
  - Inorder?
  - Postorder?



Examples

- In what order will the values of the nodes be printed if we print the tree by traversing it:
  - Preorder? 0, 1, 2, 6, 7 .
  - Inorder? 1, 0, 6, 2, 7.
  - Postorder? 1, 6, 7, 2, 0.



Recursive Tree Traversal

```
void traverse_preorder(link h)
{
   if (h == NULL) return;
   do_something_with(h);
   traverse(h->l);
   traverse(h->r);
}
```

```
void traverse_inorder(link h)
{
   if (h == NULL) return;
   traverse(h->l);
   do_something_with(h);
   traverse(h->r);
}
```

```
void traverse_postorder(link h)
{
   if (h == NULL) return;
   traverse(h->l);
   traverse(h->r);
   do_something_with(h);
}
```

Recursive Examples

Counting the number of nodes in the tree:

```
int count(link h)
{
   if (h == NULL) return 0;
   int c1 = count(h->left);
   int c2 = count(h->right);
   return c1 + c2 + 1;
}
```

Computing the height of the tree:

```
int height(link h)
{
   if (h == NULL) return -1;
   int u = height(h->left);
   int v = height(h->right);
   if (u > v) return u+1;
   else return v+1;
}
```

Recursive Examples

Printing the contents of each node:

(assuming that the items in the nodes are characters)

```
void printnode(char c, int h)
{
    int i;
    for (i = 0; i < h; i++) printf("  ");
    printf("%c\n", c);
}

void show(link x, int h)
{
    if (x == NULL) { printnode("*", h); return; }
    printnode(x->item, h);
    show(x->l, h+1);
    show(x->r, h+1);
}
```

Recursive Graph Traversal

- Recursive functions are also frequently used to traverse graphs.
- When traversing a tree, it is natural to start at the root.
- When traversing a graph, we must specify the node with start from.
- In the following examples we will assume that we represent graphs using adjacency lists.

Reminder: Defining a Graph Using Adjacency Lists

```
typedef struct struct_graph * graph;

struct struct_graph
{
   int number_of_vertices;
   list * adjacencies;
};
```

Graph Traversal - Graph Search

- Overall, we will use the terms **"graph traversal"** and **"graph search"** almost interchangeably.
- However, there is a small difference:
  - "Traversal" implies we visit every node in the graph.
  - "Search" implies we visit nodes until we find something we are looking for.
- For example:
  - A node labeled "New York".
  - A node containing integer 2014.

Graph Search in General

- GraphSearch(graph, starting_node)
  - Initialize list to_visit to a list with starting_node as its only element.
  - While(to_visit is not empty):
    - Remove a node N from list to_visit.
    - "Visit" that node.
    - If that node was what we were looking for, break.
    - Add the children of that node to the end of list to_visit.
- The pseudocode is really a template.
- It does not specify what we really want to do.
- To fully specify an algorithm, we need to better define what each of the red lines.

Graph Search in General

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    - "Visit" that node.
    - If that node was what we were looking for, break.
    - Add the children of that node to the end of list to_visit.
- Depending on what we specify in those lines, this template can produce a wide variety of applications:
  - Printing each node of the graph.
  - Driving directions.
  - The best move for a board game like chess.
  - A solution to a mathematical problem…

Specifying Graph Search Behavior

- GraphSearch(graph, starting_node)
  - Initialize list to_visit to a list with starting_node as its only element.
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    - If that node was what we were looking for, break.
    - Add the children of that node to the end of list to_visit.
- What do we do when visiting a node?
- Whatever we want. For example:
  - Print the contents of the node.
  - Use the contents in some computation (min, max, sum, ...).
  - See if the node has a value we care about ("New York", 2014, ...).
  - These are all reasonable topics for assignments/exams.

Specifying Graph Search Behavior

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- Inserting children of a node to the to_visit list:
- We have a choice: insert a child even if it already is included in that list, or not?
  - In some cases we should not. Example: ???
  - In some cases we should, but we may not see such cases in this course.

Specifying Graph Search Behavior

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- Inserting children of a node to the to_visit list:
- We have a choice: insert a child even if it already is included in that list, or not?
  - In some cases we should not. Example: printing each node.
  - In some cases we should, but we may not see such cases in this course.

Specifying Graph Search Behavior

- GraphSearch(graph, starting_node)
  - Initialize list to_visit to a list with starting_node as its only element.
  - While(to_visit is not empty):
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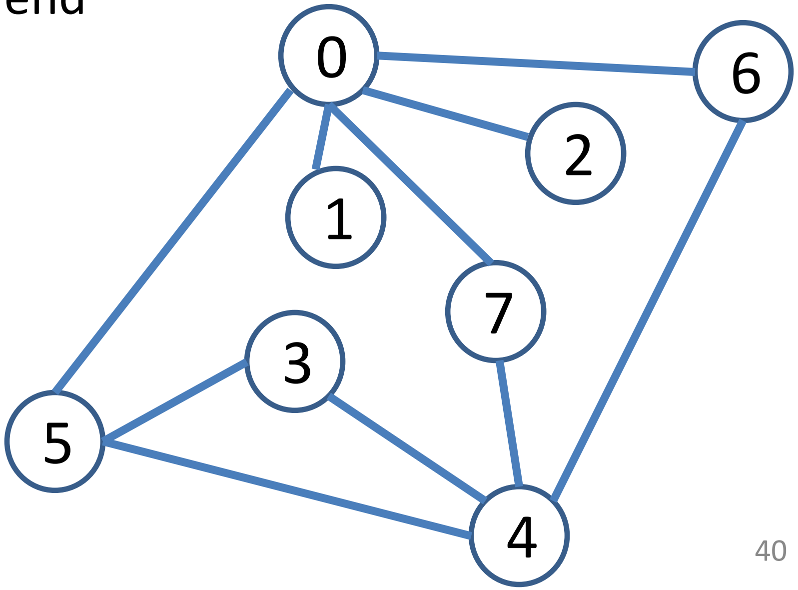
    - "Visit" that node.
    - If that node was what we were looking for, break.
    - Add the children of that node to the end of list to_visit.
- **Most important question** (for the purposes of this course):
  - Removing a node from list to_visit: **Which node**? The first, the last, some other one?
- The answer has **profound** implications for time complexity, space complexity, other issues you may see later or in other courses…

Depth-First Search

- DepthFirstSearch(graph, starting_node)
  - Initialize list to_visit to a list with starting_node as its only element.
  - While(to_visit is not empty):
    - Remove the last node N from list to_visit.
    - "Visit" that node.
    - If that node was what we were looking for, break.
    - Add the children of that node to the end of list to_visit.
- In depth-first search, the list of nodes to visit is treated as a LIFO (last-in, first-out) queue.
- DepthFirstSearch(graph, 5):
- In what order does it visit nodes?



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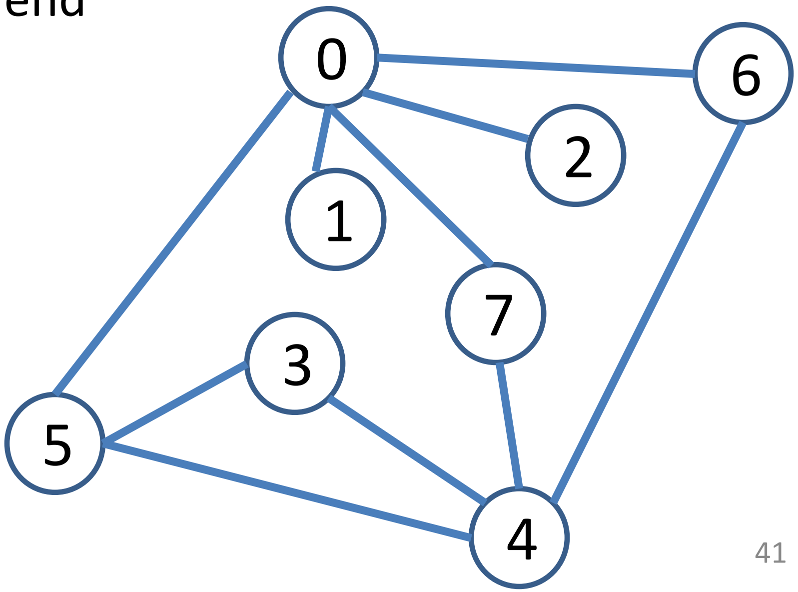
    - If that node was what we were looking for, break.
    - Add the children of that node to the end of list to_visit.
- DepthFirstSearch(graph, 5):
- In what order does it visit nodes?
- The answer is not unique.
  - One possibility: 5, 4, 3, 7, 0, 1, 2, 6.
  - Another possibility: 5, 3, 4, 7, 0, 6, 1, 2.
  - Another possibility: 5, 0, 6, 4, 3, 7, 1, 2.



Depth-First Search

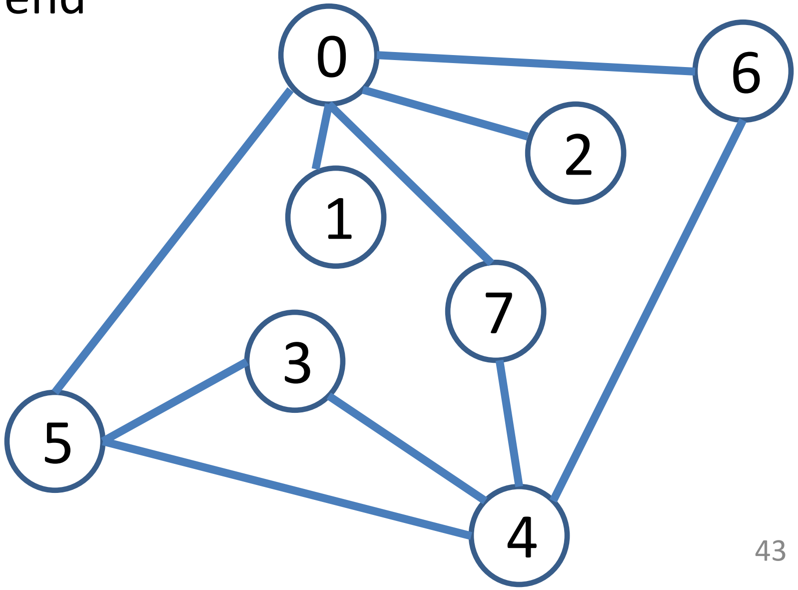
```
void depth_first(Graph g, int start)
{
   int * visited = malloc(sizeof(int) * g->number_of_vertices);
   int i;
   for (i = 0; i < g->number_of_vertices; i++)  visited[i] = 0;
   depth_first_helper(g, start, visited);
}

void depth_first_helper (Graph g, int k, int * visited)
{  link t;
   do_something_with(k);   // This is just a placeholder.
   visited[k] = 1;
   for (t = listFirst(g->adjacencies[k]); t != NULL; t = t->next)
      if (!visited[linkItem(t)])    depth_first_helper(g, linkItem(t), visited);
}
```

This code assumes that each link item is an int.

Note: no need to explicitly maintain a list of nodes to visit.

Breadth-First Search

- BreadthFirstSearch(graph, starting_node)
  - Initialize list to_visit to a list with starting_node as its only element.
  - While(to_visit is not empty):
    - Remove the first node N from list to_visit.
    - "Visit" that node.
    - If that node was what we were looking for, break.
    - Add the children of that node to the end of list to_visit.
- In breadth-first search, the list of nodes to visit is treated as a LIFO (last-in, first-out) queue.
- BreadthFirstSearch(graph, 5):
- In what order does it visit nodes?



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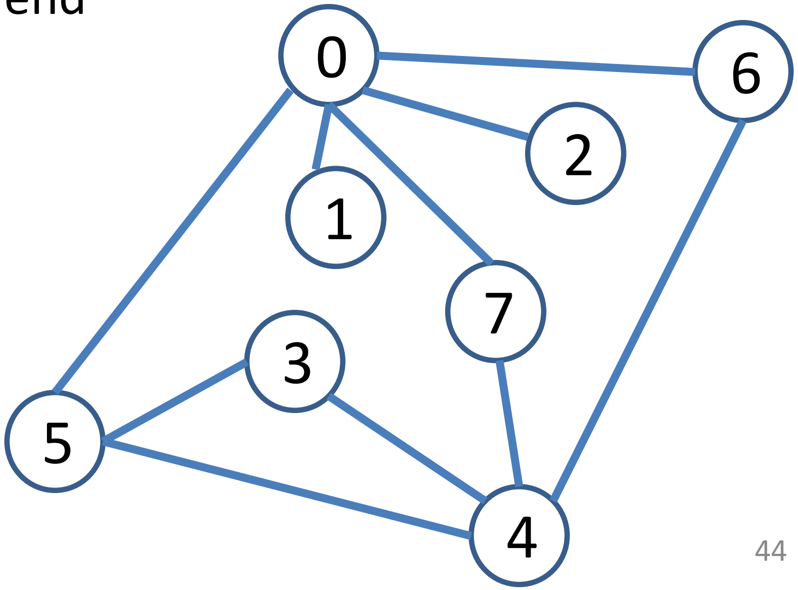
- BreadthFirstSearch(graph, 5):
- In what order does it visit nodes?
- The answer is not unique.
  - One possibility: 5, 4, 3, 0, 7, 1, 2, 6.
  - Another possibility: 5, 3, 4, 0, 7, 6, 1, 2.
  - Another possibility: 5, 0, 4, 3, 6, 1, 2, 7.



Breadth-First Search

This pseudocode uses the textbook's implementation of queues.

```
void breadth_first(Graph g, int k)
{
  int i;   link t;
  int * visited = malloc(sizeof(int) * g->number_of_vertices);
  for (i = 0; i < g->number_of_vertices; i++)  visited[i] = 0;
  QUEUEinit(V);   QUEUEput(k);
  while (!QUEUEempty())
    if (visited[k = QUEUEget()] == 0)
    {
      do_something_with(k);   // This is just a placeholder.
      visited[k] = 1;
      for (t = g->adjacencies[k]; t != NULL; t = t->next)
      if (visited[linkItem(t)] == 0)   QUEUEput(linkItem(t));
    }
}
```

Note

- The previous examples should be treated as very detailed C-like pseudocode, not as ready-to-run code.
- We have seen several different implementations of graphs, lists, queues.
- To make the code actually work, you will need to make sure it complies with specific implementations.