Alpha-Beta Search

```
function AlphaBetaDecision(state) returns an action
    v = MaxValue(state, -infinity, +infinity)
    return the a in Actions(state) leading to a successor state with utility v.


function MaxValue(state, alpha, beta) returns a utility value
    if TerminalTest(state) then return Utility(state)
    v = -infinity
    for  a, s in Successors(state) do
        v = Max(v, MinValue(s, alpha, beta))
        if v >= beta  then return v
        alpha = Max(alpha, v)
    return v


function MinValue(state, alpha, beta) returns a utility value
    if TerminalTest(state) then return Utility(state)
    v = +infinity
    for  a, s in Successors(state) do
        v = Min(v, MaxValue(s, alpha, beta))
        if v <= alpha  then return v
        beta = Min(beta, v)
    return v
```

MAX   A1 ~~3~~ 3

MIN   B1 3   B2 ≤ 2   B3 ~~≥ 14~~ ~~≤ 5~~ 2
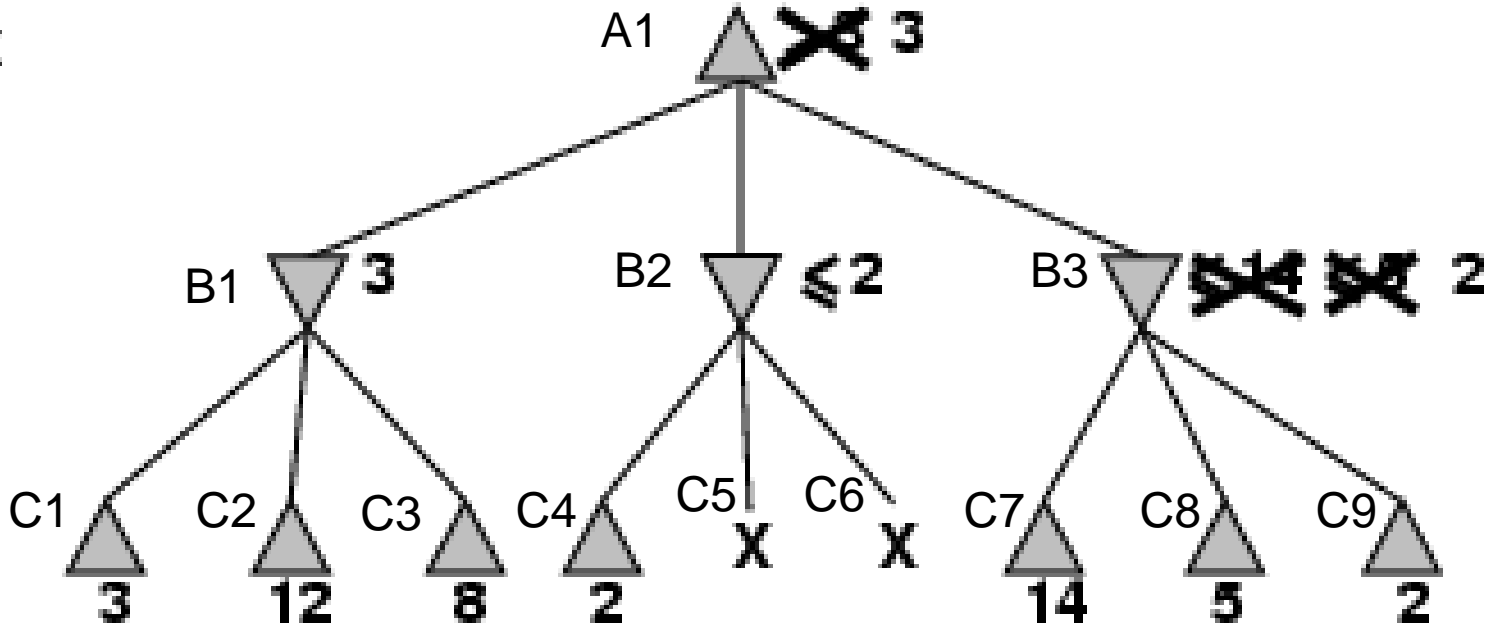
C1   C2   C3   C4   C5 X   C6 X   C7   C8   C9

3   12   8   2   14   5   2

- MaxValue(A1,-infinity,+infinity)

    TerminalState(A1)? No

    v = -infinity

    Successors(A1) = {B1, B2, B3}

    s = B1

    MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

- MinValue(B1, -infinity, +infinity)

- MaxValue(A1,-infinity,+infinity)
  - TerminalState(A1)? No
  - v = -infinity
  - Successors(A1) = {B1, B2, B3}
  - s = B1
  - MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

- MinValue(B1, -infinity, +infinity)
  - TerminalState(B1)? No
  - v = +infinity
  - Successors(B1) = {C1, C2, C3}
  - s = C1
  - MaxValue(s, alpha, beta) = MaxValue(C1, -infinity, +infinity) = 3

- MaxValue(A1,-infinity,+infinity)

  TerminalState(A1)? No

  v = -infinity

  Successors(A1) = {B1, B2, B3}

  s = B1

  MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

- MinValue(B1, -infinity, +infinity)

  TerminalState(B1)? No

  v = +infinity

  Successors(B1) = {C1, C2, C3}

  s = C1

  MaxValue(s, alpha, beta) = MaxValue(C1, -infinity, +infinity) = 3

  v = Min(v, 3) = Min(+infinity, 3) = 3

  v <= alpha? 3 <= -infinity? No

- MaxValue(A1,-infinity,+infinity)

    TerminalState(A1)? No

    v = -infinity

    Successors(A1) = {B1, B2, B3}

    s = B1

    MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

- MinValue(B1, -infinity, +infinity)

    TerminalState(B1)? No

    v = +infinity

    Successors(B1) = {C1, C2, C3}

    s = C1

    MaxValue(s, alpha, beta) = MaxValue(C1, -infinity, +infinity) = 3

    v = Min(v, 3) = Min(+infinity, 3) = 3

    v <= alpha? 3 <= -infinity? No

    beta = Min(beta, v) = min(+infinity, 3) = 3

- MaxValue(A1,-infinity,+infinity)

  TerminalState(A1)? No

  v = -infinity

  Successors(A1) = {B1, B2, B3}

  s = B1

  MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

- MinValue(B1, -infinity, +infinity)

  TerminalState(B1)? No

  v = +infinity

  Successors(B1) = {C1, C2, C3}

  s = C2

  MaxValue(s, alpha, beta) = MaxValue(C2, -infinity, 3) = 12

  v = Min(v, 12) = Min(3, 12) = 3

  v <= alpha? 3 <= -infinity? No

  beta = Min(beta, v) = min(3, 3) = 3

- MaxValue(A1,-infinity,+infinity)

    TerminalState(A1)? No

    v = -infinity

    Successors(A1) = {B1, B2, B3}

    s = B1

    MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

state = A1
alpha = -infinity
beta = +infinity
v = -infinity
s = B1

- MinValue(B1, -infinity, +infinity)

    TerminalState(B1)? No

    v = +infinity

    Successors(B1) = {C1, C2, C3}

    s = C3

    MaxValue(s, alpha, beta) = MaxValue(C3, -infinity, 3) = 8

    v = Min(v, 8) = Min(3, 12) = 3

    v <= alpha? 3 <= -infinity? No

    beta = Min(beta, v) = min(3, 3) = 3

state = B1
alpha = -infinity
beta = 3
v = 3
s = C3

- MaxValue(A1,-infinity,+infinity)
    - TerminalState(A1)? No
    - v = -infinity
    - Successors(A1) = {B1, B2, B3}
    - s = B1
    - MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity)

state = A1
alpha = -infinity
beta = +infinity
v = -infinity
s = B1

- MinValue(B1, -infinity, +infinity)
    - TerminalState(B1)? No
    - v = +infinity
    - Successors(B1) = {C1, C2, C3}
    - s = C3
    - MaxValue(s, alpha, beta) = MaxValue(C3, -infinity, 3) = 8
    - v = Min(v, 8) = Min(3, 12) = 3
    - v <= alpha? 3 <= -infinity? No
    - beta = Min(beta, v) = min(3, 3) = 3

    - return v: returns 3

state = B1
alpha = -infinity
beta = 3
v = 3
s = C3

- MaxValue(A1,-infinity,+infinity)
    TerminalState(A1)? No
    v = -infinity
    Successors(A1) = {B1, B2, B3}
    s = B1
    MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity) = 3
    v = Max(v, MinValue(B1, -infinity, +infinity)) = Max(-infinity, 3) = 3

state = A1
alpha = -infinity
beta = +infinity
v = 3
s = B1

- MaxValue(A1,-infinity,+infinity)

  TerminalState(A1)? No

  v = -infinity

  Successors(A1) = {B1, B2, B3}

  s = B1

  MinValue(s, alpha, beta) = MinValue(B1, -infinity, +infinity) = 3

  v = Max(v, MinValue(B1, -infinity, +infinity)) = Max(-infinity, 3) = 3

  v >= beta? 3 >= +infinity? No

  alpha = Max(alpha, v) = Max(-infinity, 3) = 3

state = A1
alpha = 3
beta = +infinity
v = 3
s = B1

- MaxValue(A1,-infinity,+infinity)

  TerminalState(A1)? No

  v = -infinity

  Successors(A1) = {B1, B2, B3}

  s = B2

  MinValue(s, alpha, beta) = MinValue(B2, 3, +infinity)

state = A1
alpha = 3
beta = +infinity
v = 3
s = B2

- MaxValue(A1,-infinity,+infinity)

  TerminalState(A1)? No

  v = -infinity

  Successors(A1) = {B1, B2, B3}

  s = B2

  MinValue(s, alpha, beta) = MinValue(B2, 3, +infinity)

  state = A1
  alpha = 3
  beta = +infinity
  v = 3
  s = B2

- MinValue(B2, 3, +infinity)

  state = B2
  alpha = 3
  beta = +infinity

- MaxValue(A1,-infinity,+infinity)

    TerminalState(A1)? No

    v = -infinity

    Successors(A1) = {B1, B2, B3}

    s = B2

    MinValue(s, alpha, beta) = MinValue(B2, 3, +infinity)

- MinValue(B2, 3, +infinity)

    TerminalState(B1)? No

    v = +infinity

    Successors(B2) = {C4, C5, C6}

    s = C4

    MaxValue(s, alpha, beta) = MaxValue(C4, 3, +infinity) = 2

- MaxValue(A1,-infinity,+infinity)

    TerminalState(A1)? No

    v = -infinity

    Successors(A1) = {B1, B2, B3}

    s = B2

    MinValue(s, alpha, beta) = MinValue(B2, 3, +infinity)

- MinValue(B2, 3, +infinity)

    TerminalState(B1)? No

    v = +infinity

    Successors(B2) = {C4, C5, C6}

    s = C4

    MaxValue(s, alpha, beta) = MaxValue(C4, 3, +infinity) = 2

    v = Min(v, 3) = Min(+infinity, 2) = 2

    v <= alpha? 2 <= 3? **YES**

    return v: returns 2