CSE5311 Design and Analysis of Algorithms

DP Exercise Problems

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1. A ski rental agency has m pairs of skis, where the height of the ith pair of skis is s_i. There are n skiers who wish to rent skis, where the height of the ith skier is h_i. Ideally, each skier should obtain a pair of skis whose height matches with his own height as closely as possible. Design and efficient algorithm to assign skis so that the sum of the absolute differences of the heights of each skier and his/her skis is minimized.

2. What is a 0-1 knapsack problem? What is a fractional knapsack problem? Give a dynamic-programming solution to the 0-1 knapsack problem that runs in $O(nW)$ time, where n is the number of items and W is the maximum weight of items that the thief can put in his knapsack. Prove that the fractional knapsack problem has the greedy strategy.

3. A child wants to construct the tallest tower possible out of building blocks. She has n types of blocks, and an unlimited supply of blocks of each type. Each type-i block is a rectangular solid with linear dimensions $<x_i,y_i,z_i>$. A Block can be oriented so that any two of its three dimensions determine the dimensions of base and the other dimension is the height. In building a tower one block may be placed on top of another block as long as the two dimensions of the base of the upper block are each strictly smaller than the corresponding base dimensions of the lower block. (Blocks oriented to have equal-sized bases can not be stacked). Design an efficient algorithm to determine the tallest tower the child can build.

4. Consider the problem of printing a paragraph on a printer. The input text is a sequence of n words of length $l_1,l_2,\ldots,l_n$, measured in characters. We want to print this paragraph neatly on a number of lines that hold a maximum of M characters each. Our criterion of "neatness" is as follows. If a given line contains words i through j and we leave exactly one space between words, the number of extra space characters at the end of the line is $M-j+i-\sum_{k=i}^{j} l_k$.
   We wish to minimize the sum, over all lines except the last, of the cubes of the numbers of extra space characters at the end of lines. Give a dynamic-programming algorithm to print a paragraph of n words neatly on a printer. Analyze the running time and space requirements of your algorithm.