String Matching Algorithms

Topics

- ❑Basics of Strings
- ❑Brute-force String Matcher
- ❑Rabin-Karp String Matching Algorithm
- ❑KMP Algorithm

In string matching problems, it is required to find the occurrences of a pattern in a text.

These problems find applications in text processing, text-editing, computer security, and DNA sequence analysis.

Find and Change **in word processing**

Sequence of the human cyclophilin 40 gene

CCCAGTCTGG AATACAGTGG CGCGATCTCG GTTCACTGCA

ACCGCCGCCT CCCGGGTTCA AACGATTCTC CTGCCTCAGC

CGCGATCTCG : DNA binding protein GATA-1

CCCGGG : DNA binding protein Sma 1

C: Cytosine, G : Guanine, A : Adenosine, T : Thymine

Text : T[1..n] of length n and Pattern P[1..m] of length m.
The elements of P and T are characters drawn from a finite alphabet set Σ.
For example Σ= {0,1} or Σ = {a,b, . . . , z}, or Σ = {c, g, a, t}.
The character arrays of P and T are also referred to as strings of characters.
Pattern P is said to occur with shift *s* in text T

if $0 \leq s \leq n-m$ and
T[s+1..s+m] = P[1..m] or
T[s+j] = P[j] for $1 \leq j \leq m$,
such a shift is called a valid shift.

The string-matching problem is the problem of finding all valid shifts with which a given pattern P occurs in a given text T.

Brute force string-matching algorithm

To find all valid shifts or possible values of s so that P[1..m] = T[s+1..s+m] ;
There are n-m+1 possible values of s.

Procedure BF_String_Matcher(T,P)

```
n ← length [T];
m ← length[P];
for s ← 0 to n-m
    do if P[1..m] = T[s+1..s+m]
        then shift s is valid
```

This algorithm takes $\Theta((n-m+1)m)$ in the worst case.

a c a a b c a c a a b c

a a b a a b

a c a a b c

a a b

a c a a b c matches

a a b

Rabin-Karp Algorithm

Let $\Sigma = \{0,1,2, . . .,9\}$.
We can view a string of k consecutive characters as representing a length-k decimal number.
Let p denote the decimal number for P[1..m]
Let t_s denote the decimal value of the length-m substring T[s+1..s+m] of T[1..n] for s = 0, 1, . . ., n-m.

t_s = p if and only if
T[s+1..s+m] = P[1..m], and s is a valid shift.

p = P[m] + 10(P[m-1] +10(P[m-2]+ . . . +10(P[2]+10(P[1]))
We can compute p in O(m) time.

Similarly we can compute t_0 from T[1..m] in O(m) time.

$$6378 = 8 + 7 \times 10 + 3 \times 10^2 + 6 \times 10^3 \qquad m = 4$$

$$= 8 + 10\,(7 + 10\,(3 + 10(6)))$$

$$= 8 + 70 + 300 + 6000$$

p = P[m] + 10(P[m-1] +10(P[m-2]+ . . . +10(P[2]+10(P[1]))

t_{s+1} can be computed from t_s in constant time.

$t_{s+1} = 10(t_s - 10^{m-1}\, T[s+1]) + T[s+m+1]$

Example : T = 314152
t_s = 31415, s = 0, m= 5 and T[s+m+1] = 2

t_{s+1}= 10(31415 –10000*3) +2 = 14152

Thus p and t_0, t_1, . . ., t_{n-m} can all be computed in O(n+m) time.
And all occurences of the pattern P[1..m] in the text T[1..n] can be found in time O(n+m).

However, p and t_s may be too large to work with conveniently.
Do we have a simple solution!!

Computation of p and t_0 and the recurrence is done modulus q.

In general, with a d-ary alphabet {0,1,…,d-1}, q is chosen such that $d \times q$ fits within a computer word.

The recurrence equation can be rewritten as

$t_{s+1} = (d(t_s - T[s+1]h) + T[s+m+1]) \bmod q$,

where $h = d^{m-1} (\bmod q)$ is the value of the digit "1" in the high order position of an m-digit text window.

Note that $t_s \equiv p \bmod q$ does not imply that $t_s = p$.

However, if t_s is not equivalent to p mod q , then $t_s \neq p$, and the shift s is invalid.

We use $t_s \equiv p \bmod q$ as a fast heuristic test to rule out the invalid shifts.

Further testing is done to eliminate spurious hits.

- an explicit test to check whether

$P[1..m] = T[s+1..s+m]$

$t_{s+1} = (d(t_s - T[s+1]h) + T[s+m+1]) \bmod q$

$h = d^{m-1} (\bmod q)$

Example :

T = 31415; P = 26, n = 5, m = 2, q = 11

p = 26 mod 11 = 4

t0 = 31 mod 11 = 9

t1 = (10(9 - 3(10) mod 11) + 4) mod 11

= (10 (9- 8) + 4) mod 14 = 14 mod 11 = 3

Procedure RABIN-KARP-MATCHER(T,P,d,q)
Input : Text T, pattern P, radix d (which is typically = $|\Sigma|$),
and the prime q.
Output : valid shifts s where P matches

1. n $\leftarrow$ length[T];
2. m $\leftarrow$ length[P];
3. h $\leftarrow d^{m-1}$ mod q;
4. p $\leftarrow$ 0;
5. $t_0 \leftarrow$ 0;
6. for i $\leftarrow$ 1 to m
7.   do p $\leftarrow$ (d$\times$p + P[i] mod q;
8. $t_0 \leftarrow$ (d$\times t_0$ +T[i] mod q;
9. for s $\leftarrow$ 0 to n-m
10.   do if p = t_s
11.   then if P[1..m] = T[s+1..s+m]
12.   then “pattern occurs with shift “ s
13.   if s < n-m
14.   then $t_{s+1} \leftarrow$ (d(t_s –T[s+1]h)+ T[s+m+1]) mod q;

Comments on Rabin-Karp Algorithm

- All characters are interpreted as radix-d digits
- h is initiated to the value of high order digit position of an m-digit window
- p and t_0 are computed in O(m+m) time
- The loop of line 9 takes Θ((n-m+1)m) time

The loop 6-8 takes O(m) time
The overall running time is O((n-m)+m)

Knuth Morris Pratt(KMP) Algorithm

Pseudocode :

```
KMP-Matcher (T, P)
    n ← length (T)
    m ← length (P)
    π ← Compute-Prefix-Function (P)
    q ← 0
    for i = 1 to n
      while q > 0 and P[q+1]  ≠ T [i] ;
          do q ← π [q]
      if P[q+1] = T [i]
        then q ← q + 1
      if q = m
        then print ``Pattern occurs
                     with shift'' (i - m)
        q ← π [q]
Compute-Prefix-Function (P)
```

```
Compute Prefix Function (P)
m ← length [P]
π[1] ← 0
k ← 0
for q ← 2 to m
    do while k > 0 and P[k+1]≠ P[q]
          do k ← π[k]
       if P[k+1] = P[q]
          then k ← k+1
       π [q] ← k
return π
```


- Given the pattern P [1..q] matches text chs T[s+1.. S+q]

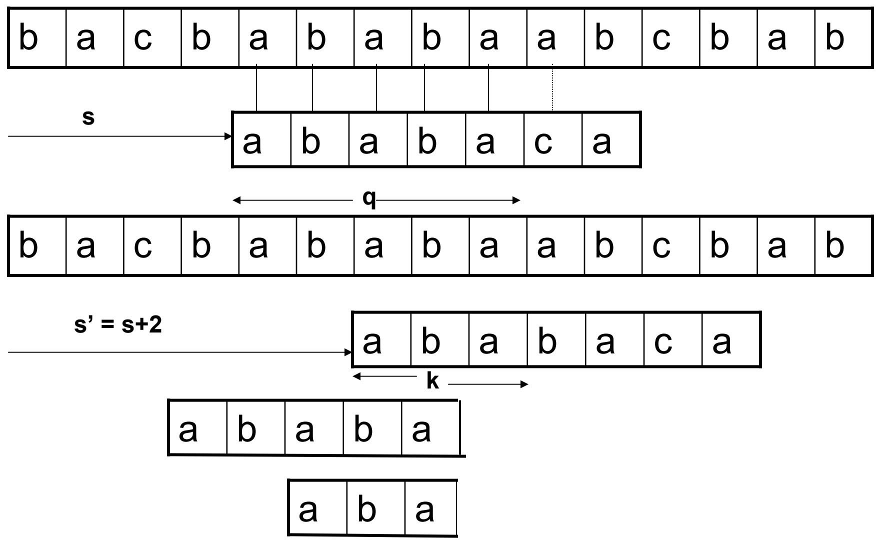
What is the least shift s’ > s such that

P[1..k] = T[s’+1, .. s’+k], s’+k = s+q

Given pattern P[1..m], the prefix function for the pattern P is the function

π : {1,2, … m } → {0,1, …m-1} such that

π [q] = max { k: k < q and P_k is a suffix of P_q



i	1	2	3	4	5	6	7	8	9	10
P[i]	a	b	a	b	a	b	a	b	c	a
π[i]	0	0	1	2	3	4	5	6	0	1

KMP algorithm (contd..)

Running time analysis of KMP yields O(m+n), because the call of the function takes O(m) time and the remainder KMP matcher algorithm takes O(n) time.

KMP is among the fastest algorithms for large sizes of P and T

Boyer Moore Algorithm

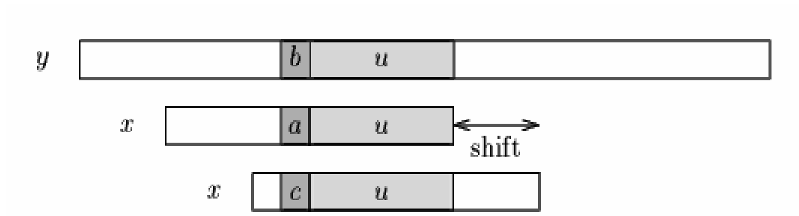
Pseudocode

```
n <--  length [T]
m <-- length [P]
∂ <-- COMPUTE-LAST-OCCURRENCE-FUNCTION(P,m,ξ)
Φ <-- COMPUTE-GOOD-SUFFIX-FUNCTION(P,m)
S <-- 0
While s ≤ n –m
    do j <-- m
     while j >0 and P[j] = T[s+j]
            do j <-- j -1
        if j =0
            then print "Pattern Occurs at shift " s
                 s <-- s+ Φ[0]
             else s <-- s + max(Φ[j],j - ∂[T[s +j]])
                  s <-- s + 1
           else s <-- s + 1
```

Boyer Moore Algorithm(contd.)

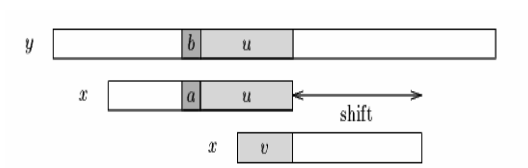
This algorithm is considered as the most efficient algorithm for most of the general applications of string matching.

- ♦ This algorithm scans the pattern from right to left
- ♦ In case of a mismatch it uses 2 pre computed functions
  (a) Good-Suffix Shift (b) Bad Character shift(occurrence shift)

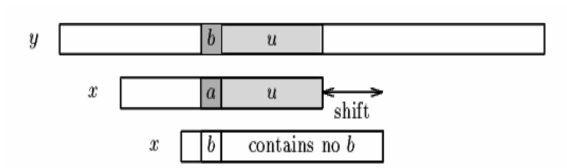


Assume that a mismatch occurs between the character *x*[*i*]=*a* of the pattern and the character *y*[*i*+*j*]=*b* of the text during an attempt at position *j*. Then, *x*[*i*+1 .. *m*-1]= *y*[*i*+*j*+1 .. *j*+*m*-1]=u and *x*[*i*]   y[*i*+*j*]. The good-suffix shift consists in aligning the segment *y*[*i*+*j*+1 .. *j*+*m*-1]=*x*[*i*+1 .. *m*-1] with its rightmost occurrence in *x* that is preceded by a character different from *x*[*i*]

Boyer Moore Algorithm(contd.)



If there exists no such segment, the shift consists in aligning the longest suffix *v* of *y*[*i*+*j*+1 .. *j*+*m*-1] with a matching prefix of *x*.



The bad-character shift consists in aligning the text character *y*[*i*+*j*] with its rightmost occurrence in *x*[0 .. *m*-2].

Boyer Moore Algorithm(contd.)

First attempt

G C A T C G C A G A G A G T A T A C A G T A C G

1

G C A G A G AG

Second attempt

G C A T C G C A G A G A G T A T A C A G T A C G

3 2 1

G C A G A G A G

Third attempt

G C A T C G C A G A G A G T A T A C A G T A C G

8 7 6 5 4 3 2 1

G C A G A G A G

Boyer Moore Algorithm(contd.)

Fourth attempt

G C A T C G C A G A G A G T A T A C A G T A C G

3 2 1

G C A G A G A G

Fifth attempt

G C A T C G C A G A G A G T A T A C A G T A C G

2 1

G C A G A G A G

Total number of character comparisons 17