Exercise Set 2

CSE 5311 Design and Analysis of Algorithms

SPRING 2007

1. Solve the following recurrence relation $T(n) = T(n-1) + n/2$, $T(1) = 1$

2. Give asymptotic upper and lower bounds for T(n) in each of the following recurrences. Assume that T(n) is constant for $n \leq 2$. Make your bounds as tight as possible, and justify your answers.
   a. $T(n) = 2T(n/2) + n^3$
   b. $T(n) = T(9n/10) + n$
   c. $T(n) = 16T(n/4) + n^2$
   d. $T(n) = 3T(n/2) + n \lg n$
   e. $T(n) = T(n-1) + \lg n$
3. Argue that the solution to the recurrence $T(n) = T(n/3) + T(2n/3) + n$ is $\Omega(n \lg n)$ by appealing to a recursion tree.

4. Use a recursion tree to solve the recurrence $T(n) = T(\alpha n) + T((1-\alpha)n) + n$, Where α is a constant in the range $0 < \alpha < 1$.

5. Prove that $T(n)$, which is defined by the recurrence relation
   $T(n) = 2T\lfloor n/2 \rfloor + 2n \log_2 n$, $T(2) = 4$,
   Satisfies $T(n) = O(n \log^2 n)$

6. Compare the following pairs of functions in terms of order of magnitude. In each case, say whether $f(n) = O(g(n))$, $f(n) = \Omega(g(n))$, and/or $f(n) = \Theta(g(n))$

	f(*n*)	**g(*n*)**
a.	$100n + \log n$	$n + (\log n)^2$
b.	$\log n$	$\log(n^2)$
c.	$n^2/\log n$	$n(\log n)^2$
d.	$(\log n)^{\log n}$	$n/\log n$
e.	$\sqrt{n}$	$(\log n)^5$
f.	$n\,2^n$	3^n