

## Problems for Quiz 2

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- 1) An undirected graph is  $k$ -connected if, for every pair of vertices  $v$  and  $w$ , there are  $k$  paths between  $v$  and  $w$  such that no vertex (except  $v$  and  $w$ ) appears in more than one path. Biconnected means 2-connected. Write a linear time (in number of vertices and edges) algorithm to find the 3-connected components of a graph.
- 2) Suppose that we have  $2n$  elements  $\langle a_1, a_2, \dots, a_{2n} \rangle$  and wish to partition them into the  $n$  smallest and the  $n$  largest. Prove that we can do this in constant additional depth after sorting  $\langle a_1, a_2, \dots, a_n \rangle$  and  $\langle a_{n+1}, a_{n+2}, \dots, a_{2n} \rangle$ .
- 3) Design a dynamic programming algorithm for the change-making problem: Given an amount  $n$  and unlimited quantities of coins of each of the denominations  $d_1, d_2, \dots, d_m$ , find the smallest number of coins that add up to  $n$  or justify that the problem does not have a solution.
- 4) Show that for an undirected graph  $G (V, E)$ , 3-coloring is NP complete. *A valid coloring of  $G$  is an assignment of colors to the vertices such that each vertex is assigned one color and no two adjacent vertices have the same color.*
- 5) Give an efficient Greedy algorithm that finds an optimal vertex cover for a tree.
- 6) Given two vertices  $s$  and  $t$  of an undirected graph, show how to use network flow to determine whether there are at least two paths between  $s$  and  $t$  that do not share any common intermediate edge (the paths are allowed to share common vertices).
- 7) Textbook, Chapter 3, Problem Number 11, page 111.
- 8) Textbook, Chapter 4, Problem Number 12, page 193.
- 9) Textbook, Chapter 6, Problem Number 10, page 321.
- 10) Textbook, Chapter 7, Solved Exercise 2, pages 412, 413.
- 11) Textbook, Chapter 7, Problem Number 12, page 420.
- 12) Textbook, Chapter 8, Solved Exercise 2, pages 502-505.