**CSE5311 Design and Analysis of Algorithms**
**Exercise Problems 1**
**01/18/07**

1. Given an array of integers $A[1..n]$, such that, for all $i$, $1 \le i < n$, we have $|A[i]-A[i+1]| \le 1$. Let $A[1] = x$ and $A[n] = y$, such that $x < y$. Design an efficient search algorithm to find j such that $A[j] = z$ for a given value $z$, $x \le z \le y$. What is the maximal number of comparisons to $z$ that your algorithm makes?

2. The input is a set $S$ containing $n$ real numbers, and a real number $x$.

   a. Design an algorithm to determine whether there are two elements of S whose sum is exactly $x$. The algorithm should run in $O(n \log n)$ time.

   b. Suppose now that the set $S$ is given in a sorted order. Design an algorithm to solve the above problem in time $O(n)$.

3. Suppose we are to find the $k$ smallest elements in a list of n elements, and we are not interested in their relative order. Can a linear-time algorithm be found when $k$ is a constant? If so give the algorithm. In either case, justify your answer.

4. Consider a set S of $n \ge 2$ distinct numbers given in unsorted order. Give an algorithm to determine two distinct numbers $x$ and $y$ ($x \ne y$) exist in the set $S$ such that $|x-y| \le |w-z|$ for all $w,z \in S$ and $w \ne z$. Your algorithm should run in O ($n \log n$) time.

5. The input is $d$ sequences of elements such that each sequence is already sorted, and there is a total of $n$ elements. Design an $O(n \log d)$ algorithm to merge all the sequences into one sorted sequence.

**<< Solutions in Class>>**